A Signaling Equilibrium Model of Intergenerational Mobility and Economic growth

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> [I]t is not a story that concludes, *Genius will out*-though Ramanujan's in the main, did. Because so nearly did events turn out otherwise that we need no imagination to see how the least bit less persistence, or the least bit less luck, might have consigned him to obscurity. In a way, then, this is also a story about social and educational *systems,* and about how they matter, and how they sometimes nurture talent and sometimes crush it. How many Ramanujans, his life begs us to ask, dwell in India today, unknown and unrecognized? And how many in America and Britain, locked away in racial or economic ghettos, scarcely aware of worlds outside of their own?

Robert Kanigel, *The Man who knew Infinity*, pp.3-4.

Issues:

- Human capital theory vs signaling theory of schooling
- existence of multiple equilibria arising from unprejudiced employer's self-fulfilling expectations in signaling models, each produce different rates of social mobility and economic growth.
- The scope of labor market practices to improve mobility and growth - one time wage contract versus quits, layoffs and promotions.
- Market signaling and Job matching: Can we get further improvement with employers of various types of wage contracts like a menu in signaling literature.
- Further improvemnt can be achieved through proper public education policies.

1 The Basic Model

Aggregate Production

$$F_t(L_t) = A_t L_t \tag{1}$$

$$A_{t+1} = A_t \left(1 + \gamma(R_t)\right) \tag{2}$$

schooling level s and innate ability $\tau \implies$ productivity level $e\left(s, \tau\right)$

Producer: Anticipates $q_t\left(e|s\right)$ and announces wage contract,

$$w_t(s;\eta) = A_t \sum_e e \cdot q_t(e|s)$$

the cost of schooling: $\theta_t(s_t, \tau_t, s_{t-1})$,

$$\max_s \; \left[w_t(s) - \theta_t(s, \tau_t, s_{t-1})\right] \tag{3}$$

optimal solution of equation (3) for agent (τ_t, s_{t-1}) by $s_t^*\left(\tau_t, s_{t-1}\right)$

Associated binary function $\chi_t(s_t, \tau_t, s_{t-1})$ which fully represents the optimal solution of (3) by

$$\chi_t(s_t, \tau_t, s_{t-1}) = \begin{cases} 1 & \text{if optimal solution for agent } (\tau_t, s_{t-1}) \text{ is } s_t \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

for Observed distribution

$$\hat{q}_t\left(e'|s\right) = \frac{\sum_{s_{t-1}} I_{e'}\left(e\left(s_t, \tau_t\right)\right) \chi_t(s_t, \tau_t, s_{t-1}) g\left(\tau_t|s_{t-1}\right) \pi_{t-1}\left(s_{t-1}\right)}{\sum_{s_{t-1}} \chi_t(s_t, \tau_t, s_{t-1}) g\left(\tau_t|s_{t-1}\right) \pi_{t-1}\left(s_{t-1}\right)} \tag{5}$$

Equilibrium transition matrix P_t:

$$p_t\left(s_{t-1}, s_t\right) = \sum_{\tau_t} \sigma_t\left(s_t, \tau_t, s_{t-1}\right) g(\tau_t|s_{t-1}) \tag{6}$$

Given π_{t-1}, the transition P_t determines π_t according to the following equation

$$\pi_t = \pi_{t-1} P_t \tag{7}$$

and π_{t-1} and $\chi_t(s_t, \tau, s_{t-1}, \eta)$ determine R_t by

$$R_t = \sum_{s_t, \tau_t, s_{t-1}} a\left(s_t, \tau_t\right) \chi_t\left(s_t, \tau_t, s_{t-1}\right) g\left(\tau_t|s_{t-1}\right) \pi_{t-1}\left(s_{t-1}\right) \tag{8}$$

the growth rate, $g(R_t)$.

Types of equilibria:

We define other kinds of equilibria relevant in the present context. A signaling equilibrium is **equal opportunity** if $s_t^*(\tau_t, s_{t-1}) = s_t^*(\tau_t, s'_{t-1}) \equiv \hat{s}\left(\tau_t\right)$, for all family background s_{t-1}, and s'_{t-1}.

1.1 Examples

Example 1 *Two levels of education and two levels of innate abilities.*

$$e(s,\tau) = \begin{cases} 1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ 2 & \text{if } s = 2, \tau = 1 \\ 3 & \text{if } s = 2, \tau = 2 \end{cases} \tag{9}$$

$$\left. \begin{array}{c} \theta(1,\tau,s_{t-1}) = 0 \; \forall \tau, s_{t-1}, \text{ and} \\ \\ \theta(2,2,2) < \theta(2,1,2) < 1 + p < \theta(2,2,1) < \theta(2,1,1) \end{array} \right] \tag{10}$$

Case 1: No mobility equilibrium.

$$q_t\,(e|s) = \begin{bmatrix} 1 & 0 \\ 0 & 1-p \\ 0 & p \end{bmatrix}$$

Employer's wage contract

$$w_t(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ 2.(1-p) + 3.p & \text{if } s_t = 2 \end{cases} \quad \text{for all } t \geq 0$$

The optimal schooling decisions $s_t^*(\tau_t, s_{t-1})$:
optimal schooling function

$$s_t^*(\tau_t, s_{t-1}) = \begin{cases} 1 & \forall \tau \in \mathcal{T} \; if \; s_{t-1} = 1 \\ 2 & \forall \tau \in \mathcal{T} \; if \; s_{t-1} = 2 \end{cases} \quad \text{for all } t \geq 0$$

The transition matrix associated with $s_t^*(.)$ *is the following:*

$$P_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \forall t \geq 0$$

The equibrium shows:

- *No intergenerational mobility.*
- *Steady-state from the beginning.*
- $R_t = \pi_0^2.p$, *and* $g(p\pi_0^2) < g(p)$, *the maximum attainable productivity growth rate for the economy.*

Case 2: Higher Mobility and higher grwoth: let $v_t \equiv \frac{p}{p\pi_{t-1}^1 + \pi_{t-1}^2}$. *Note that* $v_t > p \; \forall t \geq 1$. *At* $t = 1, v_1$ *is known. The cost function also satisfies the condition:* $p < \theta(2,2,1) < v_1 < \theta(2,1,1)$.
Employer anticipates

$$\overline{q}_t\,(e|s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 - v_t \\ 0 & v_t \end{bmatrix} \quad \text{for all } t \geq 1 \tag{11}$$

and offers wage contract,

$$\overline{w}(s_t) = \begin{cases} 1 & \textit{if } s_t = 1 \\ 2.(1 - v_t) + 3.v_t & \textit{if } s_t = 2 \end{cases}$$

Given the above wage schedule, the original $s_t^*(\tau_t, s_{t-1})$:

$$s_t^*(\tau_t, s_{t-1}) = \begin{cases} 1 & \textit{if } \tau = 1 \textit{ and } s_{t-1} = 1 \\ 2 & \textit{otherwise} \end{cases} \quad \textit{for all } t \geq 0$$

The transition matrix

$$\overline{P}_t = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}$$

The equibrium shows:

- *Intergenerational mobility up to some time* t_0*, with* $\pi^2_{t_0} > \pi^2_0$
- $R_t = \pi^2_{t_0}.p$ *for all* $t > t_0$ *, steady-state grwoth rate,* $g\left(\pi^2_{t_0}.p\right) > g(p\pi^2_0)$*(previous one) but still* $< g(p)$*, the maximum attainable productivity growth rate for the economy.*

1.2 Quits, layoffs, and promotion

A signaling equilibrium is ability separating if whenever $\tau_t \neq \tau'_t$, $s^*_t(\tau_t, .) \neq s^*_t(\tau'_t, .)$.

Employer observes an output function $Y(s, \tau)$ (which in our simple case is $e(s, \tau)$). The employer can offer an wage contract, initial offer w_0 and after observing realized output $Y(s, \tau)$ the worker gets promotion with a wage w^* if it is $> c$, otherwise he gets w_* in the second phase (no promotion). One can derive the equibrium values ofr c, w_0, w_*, and w^*.

The following result can be easily established.

Proposition 2 *The signaling equilibrium with lay-offs, quits and promotions is ability separating.*

2 Labor Market Signaling and Job Matching

Consider two sectors, $\eta = 1$ and 2 and sector 2 is research oriented contribution to knowledge creation is higher if a more talented worker with higher education works in this section.