Signaling Equilibrium, Intergenerational Social Mobility and Long Run Growth *

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Abstract

This paper provides a model of intergenerational social mobility and economic growth, in which innate ability of workers, and the type of their education and jobs determine the rate of technological progress and social mobility. The innate ability and hence productivity level of an individual is private knowledge. Education not only increases productivity level, more so for the higher ability individuals, it also acts as a signaling device for one's innate productive ability for the purpose of job matching in the labor market. It is shown that in economies with one-time non-renegotiable wage contracts, there are generally multiple signaling equilibria, all being far away from generating the maximum attainable rate of social mobility and economic growth. There are no natural economic grounds that can guide to select a particular equilibrium. Various labor market practices such as quit, layoffs and promotions based on worker's or employer's subjective assessment of on-the-job realized productivity, or explicit wage contracts contingent on some publicly observed noisy measurement of realized productivity, can improve some of the inefficiencies, and hence increase the rate of economic growth and social mobility. The remaining inefficiencies, however, can only be removed by intervening in the education system. The paper analyzes briefly a few education systems, and within the dual private-public education system, the paper examines the role of school vouchers or subsidies to the children of poorer family backgrounds in improving the rate of economic growth and social mobility.

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> [I]t is not a story that concludes, *Genius will out*-though Ramanujan's in tso nearly did events turn out otherwise that we need no imagination to see how the least bit less persistence, or the least bit less luck, might have consigned him to obscurity. In a way, then, this is also a story about social and educational *systems,* and about how they matter, and how they sometimes nurture talent and sometimes crush it. How many Ramanujans, his life begs us to ask, dwell in India today, unknown and unrecognized? And how many in America and Britain, locked away in racial or economic ghettos, scarcely aware of worlds outside of their own?
>
> Robert Kanigel, *The Man who knew Infinity*, pp.3-4.

A Signaling Model of Parental Preschool Investment and Social Mobility

1 Introduction

This heterogeneity in innate ability may make a big difference in the process of knowledge creation; while average level of education is important, the distribution of education among the workers and the distribution of jobs among workers become a more important determinant of the rate of knowledge creation. For instance, suppose workers with high innate ability get educated in technical areas and work in jobs that are suitable to generate basic scientific and technological knowledge, and workers of somewhat lesser talent may get educated in engineering schools and work in jobs that generate applied knowledge, suitable for adopting basic scientific knowledge to industrial production purposes, and so on for other talent levels.

The well functioning of the labor market and school system is a critical determinant of social mobility and hence to total factor productivity growth. To see this simply, assume that the innate talent of an individual is independent of his or her family background.[1] Suppose the family background also influences individual schooling choice: the children with poorer

[1] There is no well established empirical evidence that the innate talent of individuals is in most part genetically inherited. The general wisdom on this is that there is probably a small correlation between the intelligence of parents and children, but the environmental factors are so dominant that it is very difficult to isolate the genetic inheritance, see Levine and Suzuki [1993].

family background have lower level of schooling (which has been widely found in empirical studies). Then it is quite clear that if talented individuals from poorer background did not obtain higher schooling, they are not tapped into the growth process. In such economies, therefore, higher social mobility through higher education of talented individuals from all backgrounds, and their assignment to the appropriate jobs in the growth enhancing modern sectors will lead to higher mobility and faster growth.

There are many barriers to social mobility and growth. We focus on two important ones – (i) asymmetric information regarding the innate ability or talent of an individual (i.e., innate ability is observed by the individual but not by anyone else); (ii) learning and earning abilities depend on preschool investment by parents and thus pn family background, and on innate ability;

To analyze the effects of these factors on social mobility and growth, I extend a Spence [1974] type signaling model of job matching and human capital investment to an overlapping generations framework.[2]

The rest of the paper is organized as follows: Section 2 describes the basic model where we assume the relationship between family background and cost of education is exogenously given. In this section we define the signaling equilibrium with one-time non-renegotiable wage contracts, and a few concepts involving the equilibrium which are used later.

2 The Basic Model

We consider an overlapping generations model in which in each period one person is born to each parent. The gender of an individual is not important for this model, and we assume for ease of presentation the male gender. We denote by τ an individual's innate ability which affect one's productivity in the workplace as well as learning in school. There is some dispute in the sociology, psychometry and economics literature as to whether τ should consist of one component, which is then generally referred to as level of intelligence, or IQ, or should it be a vector allowing individuals to vary in their ability for various skills, such as verbal skill, mathematical or logical skills.[3] To keep our analysis manageable, however,

[2] There are alternative models of intergenerational social mobility, see for instance, Becker and Tomes [1979, 1986], and Loury [1981]. These models do not feature asymmetric information regarding innate productive ability of workers.

[3] The arguments in the first strand is based on statistical analyses which show that generally many test scores exhibit the presence of a common factor, known as g-loaded factor which has been found to be a good

we assume that τ is one dimensional, and children are born with a talent type represented by a finite set of discrete ordinal numbers, $\mathcal{T} = \{1, 2, ..., \hat{\tau}\}$. In $\mathcal{T}$, a higher number denotes a greater talent. The probability that a parent has a child of talent $\tau \in \mathcal{T}$ is $g(\tau)$.[4]

Another controversy which drew a lot of public debate is whether children's innate ability is genetically inherited from parent's innate ability. The general consensus is that there is some positive correlation. Although much of what we demonstrate will be valid for this general case, for ease of exposition and calculation, we assume that the probability mass function $g(\tau)$ does not depend on parent's talent type.

The talent type of an individual is private information, observed only by him and by no one else. Individuals, however, can choose an education level and its quality to signal his talent type. We denote a signal by a one[5] dimensional variable s, which we can view as quality adjusted number of years of schooling, a higher number representing either a better quality or higher schooling level. We further assume that the set of education levels, $\mathcal{S}$ is discrete and finite and is given by the ordinal numbers $\mathcal{S} = \{1, 2, ..., \hat{s}\}$, with a higher number representing a higher education level.[6]

We can think of each signal in $\mathcal{S}$ as a social class, social status, social rank according to earnings[7] or we can think of it simply as an occupation, depending on how we define the signals in $\mathcal{S}$. Furthermore, we assume that the parents' socio-economic status $s \in \mathcal{S}$ summarizes their children's family background or "environment". In each period the active

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predictor of many socio-political-economic outcomes, especially in the labor market success and educational attainment. (See, for instance, Herrnstein and Murray [1994], Gottfredson [1997]). There are many economic studies which find the opposite. (See, for instance, Heckman's critique of the Herrnstein and Murray book for arguments and references, and see Roy's [1951] model of earnings function that uses a multi-dimensional vector for ability).

[4]There are other controversies regarding talent, ability and intelligence. Some believe that one is born with a fixed level of intelligence, and training and environment has no effect on intelligence. Others do not agree with it, and believe that ability, intelligence and talent could be improved to some extent with better environment and training. Some believe that intelligence or innate ability is fixed when one is born, and less intelligent people can learn and do complex things that we face in our everyday life, in school curricula, and in modern jobs, except that they might take longer, and thus less productive; this is the view we take in this paper.

[5]Realistically, s is a vector, $s = (s^1, s^2, s^3)$, where s^1 represents the number of years of schooling, s^2 represents major specialization, such as Engineering, Medical Science, Physics, Chemistry, Business Administration, and other general subjects, and s^3 may represent quality of education, which is generally associated with the quality of the school from which s^1, and s^2 are obtained, for instance private, public, the average of the top 25% SAT scores of the school, student faculty ratio, and a host of such variables of the school (see (Daniel, Black and Smith [1995], and Heckman, Layne-Farrar and Todd [1995] for some of these variables in their empirical studies on school quality using NLSY data).

[6]The general practice in the human capital literature is, however, to treat S as continuous variable, more realistically it is a discrete set.

[7]We will see later that earnings are functions of $s \in \mathcal{S}$.

members of the society will belong to one of the groups in $\mathcal{S}$. We are interested in modeling the intergenerational mobility of families across the groups in $\mathcal{S}$, without referring it as social mobility, class mobility or earnings mobility since they all coincide in our model. More specifically, let the probability mass function of the population in period t over the set of signals $\mathcal{S}$ be denoted as $\pi_t = \left(\pi_t^1, ..., \pi_t^{\widehat{s}}\right)$, $t \geq 0$. The economy begins at time $t = 1$ with an adult population whose parents' socio-economic status is distributed as $\pi_0 = \left(\pi_0^1, ..., \pi_0^{\widehat{s}}\right)$. The mobility matrix P_t consisting of transition probabilities of an individual born in a family background s_{t-1} will move to the family background s_t, in period t, for all values of $s_{t-1}, s_t \in \mathcal{S}$. Thus given π_0 and P_1, the distribution of population in period $t = 1$ is determined. The same process rolls over in the next period, and repeats until the end of time.

In what follows we provide an economic model of how individuals decide their human capital investment, and how they get matched with jobs in the labor market. Given the initial income distribution π_0, these two factors determine the mobility matrix P_t and the dynamics of the income distribution π_t over time, the nature of the intergenerational mobility, and how they affect the rate of technological progress and the growth rate of earnings. Our emphasis is to examine the effect of asymmetric information on these decisions and the general equilibrium effect of these decisions. We begin with modeling of the production sector.

2.1 Production sector

There are some controversy as to whether years of education is a significant determinant of earnings, or is it that ability is the main determinant of earnings, and education just picks up the effect of ability (see Griliches and Mason [1972] and for more recent references, see Willis [1986]. It has been found that the effect of education on earnings is somewhat lower after ability is controlled for, but it is not insignificant. Much of these empirical studies are carried out in the Mincer [1958] earnings function framework, where they estimate market wage rate $w = \phi(s, \tau)$, where s is the number of years of schooling, and τ is an ability measure.[8] It is important to distinguish between an earnings function as above and the "productivity function" in order to understand where signaling theory of educational investment differs from human capital investment theory. *Productivity function*, $e(s, \tau)$, is the number of efficiency unit of labor that a worker with schooling level s and innate ability

[8] The original Mincer earnings function also includes an experience variable, x, which is generally taken to be number of years of work experience. Since it is not relevant for our issues, we drop it.

τ is equivalent to. The *earnings function* $w = \phi(s, \tau)$ is what the labor with education level s and ability τ gets paid in the market.

Let L_t be the total labor in efficiency units used in the production process. To simplify matters and without loss of much generality, we assume that the aggregate production in period t is represented by the following linear function:

$$F_t(L_t) = A_t L_t \tag{1}$$

where A_t is the total factor productivity parameter in period t.

In our model A_t is endogenously determined as follows: we presume that talented workers with higher education and working in higher up jobs in $\mathcal{H}$ can create more basic knowledge or new ideas about how to produce and distribute old products or new products cheaply. This basic knowledge is assumed to benefit future generations. More specifically, let $a(s, \tau, \eta)$ be the amount of basic scientific and engineering spillover knowledge created by a worker with education level s and innate ability τ when matched with the employer η. Let R_t denote the aggregate flow of spillover knowledge in period t in the economy, aggregated over all s, τ and η in the population. A_t evolves over time according to:

$$A_{t+1} = A_t \left(1 + \gamma(R_t)\right) \tag{2}$$

where $\gamma(R_t)$ is the growth rate of productivity level, assumed to be a time invariant function. If $R_t = 0$, $\gamma(R_t) = 0$ and γ may be assumed to be an increasing function as for instance, $\gamma(R_t) = R_t^{\mu}, \mu > 0$.

I assume that the production sector is competitive; the producer is risk neutral and he treats A_t as an externality when making his decisions. In each period $t \geq 1$, the producer's role is to announce a wage schedule $\omega_t(s_t)$ for hiring purposes. He observes the education level s_t of a worker but not his talent type τ_t. The employer η_t holds a subjective belief about the conditional probability distribution of the productivity level $e\left(s_t, \tau_t\right)$ of an worker given his observed education level. We denote it by

$$q_t\left(e|s_t\right) = \text{Probability}\{e|s_t\}, e \in \mathcal{E}, s_t \in \mathcal{S} \tag{3}$$

Let $\omega_t(s_t; \eta_t)$ be the wage profile that the producer η_t announces. Perfect competition, and expected profit maximization imply that

$$\begin{aligned} \omega_t(s_t) &= A_t \textstyle\sum_{e \in \mathcal{E}} e \cdot q_t(e|s_t) \\ &\equiv A_t w_t(s_t), \text{ say} \end{aligned} \tag{4}$$

where $w_t(s_t) \equiv \sum_{e \in \mathcal{E}} e \cdot q_t\left(e|s_t\right)$. Notice that $w(.)$ depends on the producer's subjective conditional probability distribution, $q_t(e|s_t)$, and dependence of $w_t(.)$ on t is through the dependence of q on t.

3 Parental Preschool investment

So far, we assumed that cost of schooling depends on one's family background which we assumed to represent children's learning environment; individuals had no control over it. This assumption that the "destiny" of children are fixed by birth (as in the "Indian caste system") might seem very restrictive. Parents care about their children's welfare. Thus, they may like to incur pre-school human capital investment so that their children have better opportunities for learning. Would this change the basic nature of the equilibrium we studied in section **??**? We examine these issues here.

We assume that the cost of producing signal s_t for an individual of talent type τ_t in period t depends on the level of parental pre-school investment h_t which is now decided by parents. We further assume that a pre-school investment of h_t on an adult of period t costs his parents $A_t h_t$ amount of t-th period resources; we assume as in the previous section that the cost of obtaining signal s_t for an adult with talent level τ_t and pre-school investment h_t is proportional to the t-th period productivity level as follows:

$$\theta_t(s_t, \tau_t, h_t) = A_t.\theta(s_t, \tau_t, h_t) \tag{5}$$

In this section, we identify an adult agent in period t by his pre-school investment h_t and talent level τ_t and **denote the agent by** (τ_t, h_t)**.** As in the previous section, the employers in period $t, t \geq 1$, form their subjective beliefs regarding the relationship between schooling level and the productivity level of workers who are in the job market, and announce a competitive wage schedule, $A_t w_t\left(s_t\right)$. A worker (τ_t, h_t) in period t takes this wage schedule $w_t\left(s_t\right)$ and his schooling cost function (5) as given and decides his own schooling level s_t, and the pre-school investment h_{t+1}on his child. We assume that the parents do not observe their children's ability when making the pre-school investment decisions, and thus h_{t+1} is not a function of his child's ability. The budget constraint of the agent (τ_t, h_t) is given by:

$$c_t = A_t\left[w_t\left(s_t\right) - \theta\left(s_t, \tau_t, h_t\right)\right] - A_{t+1}h_{t+1} \geq 0 \tag{6}$$

We assume that agent (τ_t, h_t)'s decisions are guided by the following Von Neumann-Morgenstern expected altruistic utility function:

$$E_{\tau_{t+11},\tau_{t+2},...}U_t\left(c_t,c_{t+1}...\right)=E_{\tau_{t+11},\tau_{t+2},...}\sum_{\iota=0}^{\infty}\beta^i u\left(c_{t+i}\right) \tag{7}$$

To keep our exposition simple, we assume that there are finite number of human capital investments choices from the set $\Xi=\left\{h^1,h^2,...,h^{\hat{h}}\right\}$. Let us denote the optimal schooling decision and optimal pre-school investment decision functions of agent (τ_t,h_t) by $\sigma_t(\tau_t,h_t)$ and $\psi_t(\tau_t,h_t)$ respectively. The corresponding decisions in binary form $\chi^{\sigma_t}\left(s_t,\tau_t,h_t\right)$ and $\chi^{\psi_t}\left(s_t,\tau_t,h_t\right)$ are defined respectively by

$$\chi^{\sigma_t}\left(s_t,\tau_t,h_t\right)=\begin{cases}1 & \text{if } s_t=\sigma_t\left(\tau_t,h_t\right)\\ 0 & \text{otherwise}\end{cases}$$

and

$$\chi^{\psi_t}\left(h_{t+1},\tau_t,h_t\right)=\begin{cases}1 & \text{if } h_{t+1}=\psi_t\left(\tau_t,h_t\right)\\ 0 & \text{otherwise}\end{cases}$$

Let us denote by ξ_t the probability distribution of the agents (τ_t,h_t) in period t, i.e., $\xi_t\left(\tau,h\right)\equiv P\left\{(\tau_t,h_t)=(\tau,h)\right\}$, is the proportion of the adult population that belong to the group $(\tau_t,h_t)=(\tau,h)$. We assume that the initial distribution ξ_0 is given. Given the distribution ξ_t of agents (τ_t,h_t) in period t, $t\geq 1$, and given above optimal schooling and pre-school investment decisions $\chi^{\sigma_t}\left(s_t,\tau_t,h_t\right)$ and $\chi^{\psi_t}\left(h_{t+1},\tau_t,h_t\right)$ in binary form for all agents (τ_t,h_t), we get the flow of spillover research knowledge R_t and distribution of agents $\xi_{t+1}(\tau_{t+1},h_{t+1})$ for the next period as follows:

$$R_t=\sum_{s_t,\tau_t,h_t}a\left(s_t,\tau_t\right)\chi^{\sigma_t}\left(s_t,\tau_t,h_t\right)\xi_t\left(\tau_t,h_t\right) \tag{8}$$

$$\xi_{t+1}\left(\tau_{t+1},h_{t+1}\right)=g\left(\tau_{t+1}\right)\sum_{\tau_t,h_t}\chi^{\psi_t}\left(h_{t+1},\tau_t,h_t\right)\xi_t\left(\tau_t,h_t\right) \tag{9}$$

The economy moves over time to produce **a feasible path** as follows: The initial distribution of agents, ξ_1 over $\mathcal{T}\times\Xi$ is given; the employers announce a wage schedule $w_1\left(s_1\right)$; each agent $(\tau_1,h_1)\in\mathcal{T}\times\Xi$ chooses an own schooling rule $s_1=\sigma_1\left(\tau_1,h_1\right)$, and a pre-school investment on his children decision rule $h_2=\psi_1\left(\tau_1,h_1\right)$, such that he has non-negative consumption given in equation (6). determines his consumption, and the binary variables $\chi^{\sigma_1}\left(s_1,\tau_1,h_1\right)$ and $\chi^{\psi_1}\left(h_2,\tau_1,h_1\right)$ that are associated respectively with the decisions $\sigma_1(.)$ and $\psi_1(.)$ are determined. The spillover knowledge R_1 in period $t=1$ and the distributions of agents for the next period $\xi_2\left(\tau_2,h_2\right)$ are determined from equations (8) and (9); this process iterates over time to get all the future quantities.

Definition 1. A **signaling equilibrium with endogenous pre-school investment** is a sequence of feasible wage schedule, schooling and pre-school investment decision functions $\{w_t(.), \sigma_t(.,.), \psi_t(.,.)\}$ and an initial distribution of population $\xi_1(\tau_1, h_1)$ such that for all $t \geq 1$,for all agents $(\tau_t, h_t) \in \mathcal{T} \times \Xi$, and given the wage schedule $w_t(s_t)$,

(a) $\sigma_t(\tau_t, h_t)$ is his optimal schooling function,

(b) $\psi_t(\tau_t, h_t)$ is his optimal pre-school investment function, and

(c) the wage schedule $w_t(s_t)$ satisfies the following self-fulfilling expectations condition:

$$w_t(s_t) = \frac{\sum_{\tau_t,h_t} e(s_t, \tau_t)\, \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t)}{\sum_{\tau_t,h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t)}$$

for all $s_t \in \mathcal{S}$ which are chosen by some agent (τ_t, h_t).

Given the optimal schooling decision $\sigma_t(\tau_t, h_t)$ and the population distribution $\xi_t(\tau_t, h_t)$ we can derive the distribution of social status s_t over $\mathcal{S}$ in period t for all $t \geq 1$ from

$$\pi_t^{s_t} = \sum_{\tau_t,h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t), \quad s_t \in \mathcal{S} \tag{10}$$

and the transition function $P_t = [p_t(s_t, s_{t+1})]_{s_t, s_{t+1} \in \mathcal{S}}$ over $\mathcal{S}$ from

$$p_t(s_t, s_{t+1}) = \frac{\sum_{\tau_{t+1},\tau_t,h_t} g(\tau_{t+1}) \cdot \chi^{\sigma_{t+1}}(s_{t+1}, \tau_{t+1}, \psi_t(\tau_t, h_t)) \cdot \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi(\tau_t, h_t)}{\sum_{\tau_t,h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi(\tau_t, h_t)} \tag{11}$$

Notice that since $w_t(.)$ is a one-one function of s_t, the above two also define the earnings distribution for each generation, and the transition matrix of intergenerational earnings mobility.

The basic question is then: How do we compute a signaling equilibrium as defined in definition 1? Could we use the recursive structure or a Markovian structure of the dynamic programming? This framework may not, however, produce the recursive structure generally used in standard neo-classical stochastic growth models . To see this notice that since there are generally multiple signaling equilibrium wage schedules in every period that we have seen in the previous section, the equilibrium wage schedule $w_t()$ may differ in two periods for the economy with everything else same if the employers use different equilibrium conditional probabilities $q(.)$ in two periods. We rule it out by assuming that employers hold the same conditional expectations and thus announce the same wage schedule when the economy consist of the same distributions of workers and their pre-school investment. Let us denote the maximized value of the von Neumann Morgenstern expected altruistic function of agent (τ_t, h_t) by $V_t(\tau_t, h_t)$. Under above assumptions, we can solve agent (τ_t, h_t)'s optimal decision problem using the following functional equation of a dynamic programming problem:

$$\tilde{V}_t(\tau_t, h_t) = \max_{h_{t+1}} \left[u\left(A_t \widehat{w}_t(h_t, \tau_t) - A_{t+1} h_{t+1}\right) + \beta E_{\tau_{t+1}} \tilde{V}_{t+1}(\tau_{t+1}, h_{t+1}) \right] \quad (12)$$

where,

$$\widehat{w}_t(\tau_t, h_t) \equiv w_t(\sigma_t(\tau_t, h_t)) - \theta(s_t(\tau_t, h_t), \tau_t, h_t)$$

We further assume that the instantaneous utility function u satisfies the property that $u(x.y) = u(x).u(y)$. Let us use a transformation $V_t(h_t, \tau_t) = \tilde{V}_t(h_t, \tau_t) / u(A_t)$.Then the problem (12) becomes

$$\begin{aligned} V_t(h_t, \tau_t) &= \max_{h_{t+1} \in \Xi} u\left(\widehat{w}_t(h_t, \tau_t) - \frac{A_{t+1}}{A_t} h_{t+1}\right) + \beta E_{\tau_{t+1}} V_{t+1}(h_{t+1}, \tau_{t+1}) \frac{u(A_{t+1})}{u(A_t)} \quad (13) \\ &= \max_{h_{t+1} \in \Xi} u\left(\widehat{w}_t(h_t, \tau_t) - (1 + \gamma(R_t)) h_{t+1}\right) + \beta u(1 + \gamma(R_t)) E_{\tau_{t+1}} V_{t+1}(h_{t+1}, \tau_{t+1}) \end{aligned}$$

Definition 2. A **Markov perfect stationary signaling equilibrium** is a wage schedule $w(s), s \in \mathcal{S}$, the optimal schooling decision function $\sigma(\tau, h)$, the optimal policy function $h_+ = \psi(\tau, h)$ acting as optimal pre-school investment function, a probability distribution ξ over $\mathcal{T} \times \Xi$, and the flow of spillover knowledge $R > 0$, satisfy the following system of equations, (14)-(19):

$$w(s) = \sum_{e \in E} e \cdot q(e|s), \quad (14)$$

for some subjective beliefs $q(e|s)$ held by the employers,

$$\sigma(\tau, h) = \arg\max_{s \in S} [w(s) - \theta(s, \tau, h)], \quad (15)$$

$$R = \sum_{s, \tau h} a(s, \tau) \chi^{\sigma}(s, \tau, h) \xi(\tau, h) \quad (16)$$

the function $h_+ = \psi(\tau, h)$ is the optimal policy function associated with the following functional equation or the Bellman equation of the dynamic programming:

$$V(\tau, h) = \max_{h_+} \left[u\left(\widehat{w}(\tau, h) - (1 + \gamma(R)) h_+\right) + \beta u(1 + \gamma(R)) E_{\tau_+} V(\tau_+, h_+) \right] \quad (17)$$

where, $\widehat{w}_t(\tau,h) \equiv w(\sigma(\tau,h)) - \theta(\sigma(\tau,h),\tau,h)$,

$$w(s) = \frac{\sum_{\tau,h} e(s,\tau)\,\chi^{\sigma}(s,\tau,h)\,\xi(\tau,h)}{\sum_{\tau,h} \chi^{\sigma}(s,\tau,h)\,\xi(\tau,h)} \tag{18}$$

and $\xi(,.,)$ is the invariant distribution of the transition matrix over $\mathcal{T} \times \Xi$ as follows:

$$\xi(\tau_+,h_+) = g(\tau_+) \sum_{\tau\in\mathcal{T},h\in\Xi} \chi^{\psi}(h_+,\tau,h))\,\xi(\tau,h) \tag{19}$$

where $\chi^h(h_+,\tau,h)$ is a binary variable taking value 1 if $h_+ = \psi(\tau,h)$ and taking value 0 otherwise.

We can use techniques from Stokey and Lucas [1990] to analyze the above functional equation and hence the nature of the stationary equilibrium for continuous state space. The following general result can be proved:

Proposition 1. The value function $V(\tau,h)$, the optimal schooling function $\sigma(\tau,h)$ and the optimal pre-school investment function $h_+ = \psi(\tau,h)$ of agent (τ,h) are all increasing functions of τ and h, for all τ, and h.

It is not trivial even to compute a Markov perfect stationary equilibrium. We can, however, use linear programming techniques to verify if a given wage schedule $w(s)$, schooling and pre-school investment functions $\sigma(\tau,h)$ and $\psi(\tau,h)$ constitute a Markov perfect stationary signaling equilibrium. We illuminate the effect that endogeneity of pre-school investment has on the nature of growth and social mobility with the basic economy that we studied in section **??**. We turn to it next.

4 A few concepts

Given an equilibrium path of mobility matrices $\{P_t\}$, the income distributions $\{\pi_t\}$, the equilibrium wage schedules $\{w_t(.)\}$, and the flow of spillover scientific knowledge $\{R_t\}$, we define the **wage growth rate due to social mobility** between period t and $t+1$ by $\gamma_w = \sum_{i,j\in S}[(w_{t+1}(j) - w_t(i))/w_t(i)]P_t(i,j)\pi_t^i$, and **wage growth rate due to total factor productivity growth** between period t and $t+1$ by $\gamma(R_t)$. Notice that the average growth rate of earnings between period t and $t+1$ is the sum of γ_w and $\gamma(R_t)$.

It is possible to have different types of signaling equilibria. A **pure pooling signaling equilibrium** is a signaling equilibrium in which all types of agents from all economic backgrounds use the same signal, i.e., $\sigma_t(\tau_t, s_{t-1})$ is independent of τ_t and s_{t-1} for all $t \geq 1$. A **strict separating equilibrium** is a signaling equilibrium in which the agents of different talent type and family background use distinct schooling levels, and jobs i.e., $\sigma_t(\tau_t, s_{t-1}) = \sigma_t(\tau'_t, s'_{t-1})$ or $\eta^*_t(\tau_t, s_{t-1}) = \eta^*_t(\tau'_t, s'_{t-1})$ if and only if $\tau_t = \tau'_t$ and $s_{t-1} = s'_{t-1}$ for all $t \geq 1$. These are the kinds of equilibria generally studied in the game theory literature.

We define other kinds of equilibria relevant to our context. An **equal opportunity signaling equilibrium** is one in which $\sigma_t(\tau_t, s_{t-1}) = \sigma_t(\tau_t, s'_{t-1}) \equiv \bar{s}_t(\tau_t)$ say, or $\eta^*_t(\tau_t, s_{t-1}) = \eta^*_t(\tau_t, s'_{t-1}) \equiv \overline{\eta}_t(\tau_t)$ say $\forall s_{t-1}, s'_{t-1} \in \mathcal{S}$, i.e., all workers of the same talent type get the same education level or get the same job, and hence get paid the same wages, no matter what their family backgrounds are. An **equal opportunity separating equilibrium** is an equal opportunity equilibrium such that $\bar{s}_t(\tau) \neq \bar{s}_t(\tau')$ or $\overline{\eta}_t(\tau) \neq \overline{\eta}_t(\tau')$ if $\tau \neq \tau'$.

In our framework, the following result is straightforward.

Proposition 2. It is impossible to have a strict separating equilibrium.

In the next section we will consider implicit contracts and show that it is possible to get strictly separating equilibrium.

It is often difficult to compute the equilibrium path of an economic system, and we often like to study the properties of stationary equilibrium, which we define as follows:

Definition 3. A **stationary signaling equilibrium** is a signaling equilibrium in which in every period t, aggregate flow of spillover knowledge R_t, subjective beliefs of the employers $q_t(.)$, and hence the wage schedule, $w_t(.)$, the transition probability matrix of the social groups, P_t, and the distribution of the social groups π_t are all independent of t.

We denote these stationary variables without a time subscript. Notice that in a stationary equilibrium, we have $\pi = \pi P$, i.e., π is an *invariant probability distribution* with respect to P, and the stationary distribution of population over the social groups are the normalized non-negative eigenvectors of the stationary transition matrix P'.

There are some controversy regarding what is the best measure of mobility corresponding to a mobility matrix, P, see Conlisk [1990]. We do not use any of those criteria, instead we propose a criterion suitable in our framework: A **growth enhancing mobility measure** $\mu(P) \equiv R/R_{\max}$, where R is the flow of spillover knowledge and $R_{\max}$ is the maximum flow of spillover knowledge attainable in the economy out of all possible assignments of jobs and education levels to workers. Thus, this measure is bounded between 0 and 1, higher number represents higher growth enhancing mobility.

In a stationary equilibrium with a mobility matrix P and an invariant probability distribution π over $\mathcal{S}$, the distribution of income is stationary over time, and hence there is no wage growth due to mobility. The positive wage growth that has been reported in many empirical studies on geographical mobility, (see Jovanovich and Moffit [1990] and Sicherman and Galor [1990], among others) is a phenomenon along the transition path.

Note that not all economies will have stationary transition probabilities, nor all equilibria will converge to a stationary equilibrium. We will also examine the nature of equilibrium dynamics for various economies. We will assume in the rest of the paper except in section **??** that $\mathcal{H}$ is a singleton set, and drop η from the arguments of all the entities.

5 An Example

To make our points clear with least technicality, we consider this simpler economy for much of our analysis. Let $\mathcal{T} = \{1,2\}$, $S = \{1,2\}$. We assume that $a(s,\tau) = 1$ if $s = 2$ and $\tau = 2$, and $a(s,\tau) = 0$ otherwise.

Whether there exists any signaling equilibrium, and if there exists one, whether there exist many equilibria some of which are Pareto superior, some of which are equal opportunity separating, some of which are growth maximizing separating, depend on the technology $e(\tau,s)$ and the cost function, $\theta(s_t,\tau_t,s_{t-1})$. We will illustrate our issues by fixing the following specification of the technology and assuming different forms for the cost function.

$$e(s,\tau) = \begin{cases} e_1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ e_2 & \text{if } s = 2, \tau = 1 \\ e_3 & \text{if } s = 2, \tau = 2 \end{cases} \tag{20}$$

An interpretation of the above is that the workers with education level 1 are unskilled workers and the talent of the unskilled workers do not affect their productivity; however, higher educated talented workers have higher productivity than higher educated not-so-talented workers.

Assume that the cost function $\theta(s_t,\tau_t.s_{t-1})$ satisfies the following:

$$\left.\begin{array}{c} \theta(1,\tau_t,s_{t-1}) = 0 \ \forall \tau_t, s_{t-1}, \text{ and} \\ \theta(2,2,2) < \theta(2,1,2) < (e_2 - e_1) + p\,(e_3 - e_2) < \theta(2,2,1) < \theta(2,1,1) \end{array}\right] \tag{21}$$

We assume the cost function $\theta(s_t, \tau_t, h_t)$ to be the same as in equation (21), with the understanding that h_t here plays the same role as s_{t-1} there, and the same productivity function. It is obvious that the optimal schooling function will be then

$$[\sigma_t(\tau_t, h_t)]_{\substack{\tau_t=1,2 \\ h_t=h_1,h_2}} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \tag{22}$$

In section **??** we saw that those economies did not exhibit social mobility in the stationary equilibrium. We want to examine if that is also the case in the present set-up with endogenous pre-school investment. To that end, we have the following:

Proposition 3. Suppose the felicity index function is linear of the form $u(c) = c$, i.e., agents are risk neutral. Suppose the schooling cost function $\theta(s, \tau, h)$ of the economy satisfies condition (21). Then there exists a stationary Markov perfect signaling equilibrium **if and only** if the pre-school investment levels h_1, h_2 satisfies condition (26) below. Whenever there exists one, in fact, there exist precisely four stationary Markov perfect equilibria, with varying degree of social mobility and economic growth and they are pareto ranked.

To see this, notice that given our assumption on schooling cost in equation (21), optimal schooling function must be of the form in equation (22). Utilizing the properties of the optimal pre-school investment function in proposition 1, it is easy to show that the there are four possible form of $\psi(\tau, h)$:

$$1)\ \psi(\tau,h) = \begin{cases} h_1 & \forall\tau \text{ and } h = h_1 \\ h_2 & \forall\tau \text{ and } h = h_2 \end{cases} \quad 2)\ \psi(\tau,h) = \begin{cases} h_1 & \forall h \text{ and } \tau = 1 \\ h_2 & \forall h \text{ and } \tau = 2 \end{cases}$$

$$3)\ \psi(\tau,h) = \begin{cases} h_1 & \text{if } \tau = 1 \text{ and } h = h_1 \\ h_2 & \forall\tau \text{ otherwise} \end{cases} \quad 4)\ \psi(\tau,h) = \begin{cases} h_2 & \text{if } \tau = 2 \text{ and } h = h_2 \\ h_1 & \text{otherwise} \end{cases}$$

Let us consider the case 1). Let us denote by $\hat{h}_i = (1 + \gamma(R))\, h_i$, and $\hat{\beta} = (1 + \gamma(R))\, \beta$. The above $\psi(\tau, h)$ to be the optimal policy function of the Bellman equation (17), the following must be satisfied for each agent (τ, h) :

For $(\tau, h) = (1, h_1)$:

$$\begin{aligned} V(1,h_1) &= w(1) - \hat{h}_1 + \hat{\beta}\left[(1-p)V(1,h_1) + pV(2,h_1)\right] ...(S1) \\ &\geq w(1) - \hat{h}_2 + \hat{\beta}\left[(1-p)V(1,h_2) + pV(2,h_2)\right] ...(S2) \end{aligned}$$

...

For $(\tau, h) = (2, h_2)$:

$$\begin{aligned} V(2,h_2) &\geq w(2) - \theta(2,2,h_2) - \hat{h}_1 + \hat{\beta}\left[(1-p)V(1,h_1) + pV(2,h_1)\right] ...(S7) \\ &= w(2) - \theta(2,2,h_2) - \hat{h}_2 + \hat{\beta}\left[(1-p)V(1,h_2) + pV(2,h_2)\right] ...(S8) \end{aligned}$$

We have above system of eight inequalities in four unknowns $V(\tau, h), \tau = 1, 2$ and $h = h_1, h_2$. If there exists a feasible solution to the above system of inequalities, then we can confirm that the postulated optimal policies $\sigma(\tau, h)$ and $\psi(\tau, h)$ and the wage schedule $w(s)$ defined in section **??** indeed constitute a Markov perfect stationary equilibrium. Notice that the above procedure is valid for verifying any other postulated equilibrium for the above basic economy, or for more general economies in which τ and h take finite number of values. For our basic economy, however, we can solve the above analytically. When τ and h take many more values, we can use an artificial linear programming to find a feasible solution to the above.

Using the equality constraints, we find the solution analytically as follows:

$$V(1,h_1) = V(2,h_1) = \frac{w(1) - (1+\gamma(R))h_1}{1-(1+\gamma(R))\beta}, \tag{23}$$

$$V(1,h_2) = \frac{w(2) - \theta(2,1,h_2) - (1+\gamma(R))h_2 + (1+\gamma(R))\beta p \Delta\theta}{1-(1+\gamma(R))\beta} \tag{24}$$

and

$$V(2,h_2) = \frac{w(2) - \theta(2,2,h_2) - (1+\gamma(R))h_2 - (1+\gamma(R))\beta(1-p)\Delta\theta}{1-(1+\gamma(R))\beta} \tag{25}$$

where $\Delta\theta = \theta(2,1,h_2) - \theta(2,2,h_2)$. Notice, however, that the above solution should also satisfy the inequality constraints of the system $(S1)-(S8)$. The constraints $(S1), S(2), (S7)$and $(S8)$, and the above solutions (23)-(25) imply that

$$h_2 - h_1 = \beta\left[w(2) - w(1) - E_\tau \theta(2,\tau,h_2)\right] \tag{26}$$

For the postulated $\sigma(\tau, h)$ and $\psi(\tau, h)$ to be an equilibrium, the economy should satisfy the linear constraint for h_1 and h_2 given in equation (26).[9] For such economies, however, we have all inequalities in $(S1)-(S8)$ as equalities, i.e., each agent would be indifferent between the pre-school investment choices h_1 and h_2. If we started with any other forms of $\psi(\tau, h)$, we will end up exactly with the above equality constraints. We still need to

[9]We must note that such economies are generically impossible to exist.

compute R and the observed conditional distribution of e given s. To that end, we must compute the transition matrix $P_1 = [p((\tau,h),(\tau_+,h_+))]\ (\tau,h),(\tau_+,h_+) \in \mathcal{T} \times \Xi$ and the associated invariant probability distribution, $\xi(\tau,h)$ over $\mathcal{T} \times \Xi$. These are given as follows:

$$P_1 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \\ 0 & 0 & 1-p & p \end{pmatrix} \begin{matrix} :(1,h_1) \\ :(2,h_1) \\ :(1,h_2) \\ :(2,h_2) \end{matrix}$$

The above has two ergodic sets: $\{(1,h_1),(2,h_1)\}$ and $\{(1,h_2),(2,h_2)\}$ Notice that The fist ergodic set corresponds to the signal class $s = 1$ and the second ergodic set corresponds to the signal class $s = 2$. Let us denote by $\xi^1 = (1-p,p,0,0)$ and $\xi^2 = (0,0,1-p,p)$ The above system has a whole range of invariant distributions, given by $\xi = \left((1-\lambda)\xi^1 + \lambda\xi^2\right), 0 \leq \lambda \leq 1$. Which particular one the system converges to, depends on the initial distribution. As before suppose that the initial proposition of population in the signal class $s = 2$ is π_0^2. Then we have $\lambda = \pi_0^2$. We compute the stationary state flow of spillover knowledge from equation (16) as $R = \pi_0^2 \cdot p$. The associated mobility matrix over the set of signals $\mathcal{S}$ can be easily seen to be

$$\tilde{P} = [p(s,s_+)]_{s,s_+=1,2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This is precisely the signaling equilibrium 1 of section **??**.

Following the above steps, we can easily compute the signaling equilibria for the other cases as given below:

$$2)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \\ 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} (1-p)^2 \\ (1-p)p \\ (1-p)p \\ p^2 \end{pmatrix}, \tilde{P} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad \begin{matrix} R = p^2 \\ \mu(P) = p \end{matrix}$$

$$3)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \\ 0 & 0 & 1-p & p \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} 0 \\ 0 \\ (1-p) \\ p \end{pmatrix}, \tilde{P} = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}, \quad \begin{matrix} R = p \\ \mu(P) = 1 \end{matrix}$$

and

$$4)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 1-p & p & 0 & 0 \\ 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} 1-p \\ p \\ 0 \\ 0 \end{pmatrix}, \tilde{P} = \begin{pmatrix} 1 & 0 \\ 1-p & p \end{pmatrix}, \quad \begin{matrix} R = 0 \\ \mu(P) = 0 \end{matrix}$$

It is also clear (and we have numerical example to assert) that when agents are not risk neutral, i.e.,the felicity index $u(c)$ is non-linear, the proposition 3 is no longer true. In fact, we have a whole non-generic subclass of economies of section **??** that had no social mobility, which still have no social mobility even after pre-school investment is endogenized by parental altruism.

Notice that equalities of $(S1) - (S8)$ imply that we can have any policy function $h_+ = \psi(\tau, h)$ as optimal policy function. This make us ponder if any of those other policy functions together with the optimal schooling function constitute another equilibrium.

6 Policies and conclusions

In this paper, we have considered a model of intergenerational social mobility and economic growth, in which innate ability of workers, and the type of their education determine the rate of technological progress and social mobility. More talented individuals with higher education can lead to higher rate of technical progress and wage growth. Moreover, higher is the rate of mobilization of these talented individuals to higher education, the higher is the rate of social mobility. Important features of our model are that the innate ability of an individual is a private knowledge, i.e., (possibly) known only to the individual and that education not only increases productivity of the individual, more so for an higher ability individual, it also acts, at least at the time of initial hiring, as a signaling for individual's innate productive ability for the purpose of job matching in the labor market.

The practical relevance of the above types of policies hinges on important empirical questions: How to estimate the schooling cost as a function of schooling level, schooling type, school quality, family background and innate ability? How much more scope remains in an economy to reduce inefficiency by developing appropriate labor market practices? Another important empirical issue in this connection is to examine if observed educational attainment and job assignments of individuals in a society are according to their innate ability, or according to their family backgrounds. The existence of multiple equilibria arising from unprejudiced employer's self-fulfilling expectations also raises important empirical questions: How to verify whether an economy is stuck with a low level equilibrium where growth rate, and social mobility are low, and how to design policies that will allow the economy to move from a low level equilibrium to an equilibrium with higher growth and social mobility?

The framework proposed here could be calibrated using real data and be used to carry out various policy analyses regarding the cost of schooling subsidies, or school vouchers, and benefits that will accrue to the society in terms higher mobility, faster economic growth, and more egalitarian income distribution. In our future work, we plan to pursue some of these issues using the NLSY (National Longitudinal Surveys of Youths) data for the US.

References

Adams, James D. [1990], ”Fundamental Stocks of Knowledge and Productivity Growth”, *J. Polit. Econ.*, 98(4)673-702.

Becker, G. [1962], ”Investment in human capital: a theoretical analysis”, *J. Polit. Econ.*,(supplement) 70:9-49.

Becker, G. and N. Tomes [1979], ”An equilibrium theory of the distribution of income and intergenerational mobility”, J. Polit. Econ.

Becker, G. and N. Tomes [1986], ”Human Capital and the rise and fall of families”, J. Polit. Econ.

Ben-Porath, Yoram [1967], ”The production of human capital and the life cycle of earnings”, *J. Polit. Econ.*, 75:352-365.

Conlisk, J. [1990], ”Monotone Mobility Matrices,” *J. Math. Sociol.*

Daniel Kermit, Dan Black and Jeffrey Smith [1995], ”College Quality and the Wages of Young Men”, (mimeo) The Wharton School of the University of Pennsylvania, Philadelhia.

Gottfredson, Linda S. [1997], ”Why g-matters: The complexity of everyday life,” *Intelligence*, vol.24(1):79-132.

Griliches, Z. and W.M.M. Mason [1972], ”Education, Income, and Ability”, J. Polit. Econ. vol.80(3):75-103.

Heckman, James J., Anne Layne-Farrar and Petra Todd [1995], ”Does Measured School Quality Really Matter? An Examination of the Earnings-Quality Relationship”, in Gary Burtless (ed), *Does Money Matter: The Link Between Schools, Achievement, and Adult Success”*, Washington DC, Brookings Institution.

Herrnstein, R.J. and C. Murray [1994], *”The Bell Curve: Intelligence and Class Structure in American Life,”*, New York:Free Press.

Jaffe, A. [1986], Technological opportunity and spillovers of R&D: Evidence from firm’s patents, profits and market value,”, *American Economic Review*, vol.76(5):984-1001.

Jovanovich, Boyan [1979], ”Job Matching and the Theory of Turnover”, *J. Polit. Econ.*, vol. 87(5), part 1:972-990.

Jovanovich, Boyan and Robert Moffit [1990], ”An Estimate of a Sectoral Model of Labor Mobility”, *J. Polit. Econ.*, 98(4):827-852.

Kreps, D. [1990], *A Course on Microeconomic Theory*, Princeton University Press, New Jersey, Princeton.

Loury, G. [1981], ”Intergenerational Transfers and the distribution of earnings”, Econometrica.

Lucas, R. [1988], ”On the mechanics of economic development”, J. Monetary Econ.

Mincer, Jacob [1958], ”Investment in human capital and personal income distribution,” *J. Polit. Econ.*, 281-302.

Raut, L.K. [1991], ”R&D spillover and productivity growth: Evidence from Indian private firms,” *Journal of Development Economics*, Vol. 48:1-23.

Raut, L.K. [1995], ”Capital Accumulation, Income Inequality, and Endogenous Fertility in an Overlapping Generations General Equilibrium Model,” *Journal of Development Economics*, Vol. 34.

Rosen, Sherwin [1977], ”Human capital: Relations between earnings and education,” in (ed) M. Intrilligator, *”Frontiers of Quantitative Economics,”* Amsterdam, North-Holland, Vol. 3B.

Roy, A.D. [1951], ”Some thoughts on the distribution of earnings”, *Oxford Economic Papers*,vol. 3:135-146.

Spence, M. [1974], ”Market Signaling: Information transfer in Hiring and Related Screening Processes”, Harvard University Press, Cambridge:Mass.

Stiglitz, Joseph E. [1975], ”The theory screening, education, and the distribution of income,” *American Economic Review*, vol.65:283-300.

Stokey, Nancy L. and Robert E. Lucas, Jr. [1989], *Recursive Methods in Economic Dynamics,”* Harvard University Press, Cambridge:Massachusetts.

Willis, Robert J. [1986], ”Wage Determinants: A survey and reinterpretation of human capital earnings functions”, in (eds) O. Ashenfelter and R. Layard, *Handbook of Labor Economics: Vol. I”*, Elsevier Science Publishers BV.

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