Early Childhood Development, Earnings Inequality and Social Mobility in an Education Signaling Model*

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Abstract

The growing income inequality has been a big concern for economists and policy makers around the world. Many factors are responsible for the observed burgeoning income inequality, such as capital outflow, relocation of jobs, declining labor union, i.e., declining bargaining power of the labor, poor regulation of financial institutions, corruption, and all-encompassing globalization. Incomes of the bottom 99 percent population in a society comes mainly from earnings, and much of the earnings inequality results from the inequality of skill formation. The children of poor socioeconomic status stays behind skill accusations as compared to their rich counterpart. In modern technology-rich economies, providing high quality education to the talented individuals and matching their jobs with the highly productive technical sector is crucial for economic growth, earnings inequality and social mobility. Because education is used as a signal for a worker's unobserved endowment of talents, its acquisition by various social groups distorts productive efficiency, lowers social mobility and increases earnings inequality. This paper provides a signaling equilibrium framework to study these issues.**JEL Classifications:** J62, O15.**Keywords:** Early Childhood Development, Social Mobility, Education Signaling Model.

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1 Introduction

The growing income inequality has been a big concern for economists and policy makers around the world. Many factors are responsible for the observed burgeoning income inequality, such as capital outflow, relocation of jobs, declining labor union, i.e., declining bargaining power of the labor, poor regulation of financial institutions, corruption, and all-encompassing globalization (Piketty, 2014; Stiglitz, 2015; Bourguignon, 2015). In most economies, incomes of the bottom 99 percent come mainly from earnings and much of the inequality in earnings results from the inequality of skill formations Autor (2014).

A growing consensus reached among educators, media writers (see for instance, Traub (2000)), researchers in sociology, psychology and education (for instance, see (Barnett, 1995; Entwisle, 1995; McCormick, 1989; Reynolds, Temple, et al., 2001; Reynolds, Ou, et al., 2018; Schweinhart et al., 1993)) and researchers in economics, (see for instance, (Currie, 2001; Currie and Almond, 2011; Currie, 2011; Duncan et al., 2010; Heckman, 2000; Heckman, Moon, et al., 2010; Heckman and Raut, 2013; Heckman and Raut, 2016; Keane and Wolpin, 1997; García et al., 2016; Raut, 2018; Raut, 2003; Maluccio et al., 2009)) is that the children of poor SES are not prepared for college because they were not prepared for school to begin with. The summarized literature below in psychology, economics and the recently emerging genetics and epigenetics of health, cognitive and noncognitive developments of children show that the most effective intervention for the children of poor SES should be introduced at the preschool stage so that these children are prepared for schools and colleges and better health.

Much of research in the last century focused on cognitive skills as the main determinant of socioeconomic behaviors, school performances and labor market outcomes. An influential but controversial line of research argues that poor parents have poor cognitive abilities and that is why they are poor; children of poor SES inherit poor cognitive abilities from their parents; thus very little can be done to improve the cognitive skills of the disadvantaged children, and hence their school performance and labor market outcomes, see Herrnstein and Murray (1994) and other references in Plomin and Deary (2015). This view has been refuted using more appropriate data, statistical techniques and microbiological evidence.

It is the interplay of personality, emotion and cognition that determines most socioeconomic behaviors. Recent research in psychology, neurobiology, experimental game theory, and economics emphasize this. A branch of the psychology literature argues and empirically validates that the emotional intelligence is an important factor in socioeconomic decisions and behaviors—

not the cognitive intelligence alone. Many definitions and measurements for emotional intelligence exit in the literature, however, the concept more relevant to our context is quoted from Mayer et al. (2004), "{[Emotional Intelligence is the] capacity to reason about emotions, and of emotions to enhance thinking. It includes the abilities to accurately perceive emotions, to access and generate emotions so as to assist thought, to understand emotions and emotional knowledge, and to reflectively regulate emotions so as to promote emotional and intellectual growth". (Bar-On, 2000; Goleman, 2009) use somewhat broader definitions by including other personality traits in their definitions. It has been found that measures based on all these different definitions are highly correlated with each other and each explains strongly many socioeconomic behaviors independent of cognitive skills, (see Chakrabarti and Chatterjea (2017) for some of these results in psychology and for a synthesis of various definitions, and (Raut, 2003; Heckman and Raut, 2016) for significant positive effects of non-cognitive skills on labor market earnings, independent of the effects of cognitive skills).

Group outcomes are generally more efficient than what individuals could do by themselves. Group activities to attain some common goal, however, require each member of the group to perform constant mind reading of the other members and evaluate how others may react to one's action. The mechanism by which one reads other's mind in a conflicting or cooperative situation is known in the psychology literature as theory of mind, a term introduced by Premack and Woodruff (1978). Doherty (2008) describes various mechanisms for the theory of mind. One who has better emotional intelligence and a better theory of mind can be more effective in a group, and can become the leader of the group. A group can have a higher level of group emotional intelligence and cognitive intelligence than another group, and can be more efficient and more productive as a result for many activities, Woolley et al. (2010). In experimental game theory such non-cognitive skills—emotional intelligence and theory of mind—play important role, (Camerer et al., 2005; Kahneman, 2013; Winter, 2014). The recent economics literature shows that non-cognitive skills such as socialization and motivation are also important for positive labor market outcomes, (Deming, 2017; Heckman and Raut, 2016; Raut, 2003; Maluccio et al., 2009).

Where are these emotional intelligence or non-cognitive skills and the cognitive skills produced? For the effect of early childhood experiences, especially mother-child interactions, on the development of the theory of mind of the child, see (Doherty, 2008; Ruffman et al., 2002). Another branch of the psychology literature, e.g. the work of Bowlby (1982), argues that affect (emotion) dysregulation which begins to form immediately after birth, especially during the first two years of age, from low quality interaction of the primary care-taker (generally the mother) with the baby can have long lasting effects on emotional development of the child in later ages. NETWORK (2004) carried a longitudinal study and found evidence for such affect dysregulation mechanisms. The emotional dysregulation also conditions cognitive developments of children. More recent neurobiology research on this phenomena confirms this, see for instance, A. N. Schore (2005) and see J. R. Schore and A. N. Schore (2008) for a survey of this line of research. When parents are incapable of producing these skills, a good preschool program can be a good substitute for it.

Around the turn of the twenty-first century, a rapidly growing microbiology literature emerged, focusing on genetic and epigenetic mechanisms of personality, emotion and cognitive developments of individuals. The twenty-century microbiology research thought full DNA mapping of human genome will be able to uncover fully the mechanism of human development. But the research in this area fell short of explaining why identical twins diverge so much in their gene expressions or phenotypes as they progress through their lives. All cells in a body starting with the single fertilized egg have the same genetic mapping (i.e.,the same DNA sequence) throughout life. It is the epigenetic (literally means on top of genetic) codes, which are influenced by the internal and external environments of the body cells, indeed determine which genes are expressed, silenced, or mutated during cell divisions, and hence determine the development of the mind and body and their health status. For instance, stress of various kinds can have effects on epigenetic reprogramming of the plasticity of various parts of the brain that perform cognitive processing, language processing, emotion or affect regulations, the size and efficiency of the working memory and the long-term memory (see McEwen and Gianaros (2011) for the effects of stress in general, Champagne et al. (2008), Hellstrom et al. (2012) for the effects of parenting practices, and Gluckman et al. (2008) for the effects of in utero environmental factors on cognitive and non-cognitive health developments). Other environmental factors such as the quality of language exposure, the presence of books, computers, musical instruments at home, the speech pattern, cognitive skills of mother and other care givers have also significant effects on the development of the neural network of the brain (i.e.,the network of dendrites, axons and synapses) specialized for language processing, creative writing or musical talents, (Mezzacappa, 2017; Murgatroyd and Spengler, 2011).

To look for microbiological evidence for the above, a number of recent neurological studies used fMRI images of brain areas for many individuals. They found that poverty has significant negative effects on the development of a child's certain brain areas that are responsible for personality, emotion and executive functions. For instance, a large scale neurological study by Noble, Houston, Brito, et al. (2015) found that family income significantly affects children's brain size, particularly in the surface area of the cerebral cortex that does most of the cognitive processing. See also their earlier study, Noble, Houston, Kan, et al. (2012) and the commentary in Balter (2015). In another large longitudinal neurological study, Hair et al. (2015) followed children starting at an early age up into their school years. They measured their scores on cognitive and academic achievements, and development of brain tissue, including gray matter of the total brain, frontal lobe, temporal lobe, and hippocampus. They found significant negative effects of poverty on developments of these brain areas and on their academic achievements.

The vast literature above suggest that early age events have many lasting effects, as I mentioned earlier. In modern technology-rich economies, providing high quality education to the talented children and matching their jobs with the highly productive technical sector is crucial for economic growth, social mobility and earnings inequality. Individuals know their own abilities but the employers do not observe them. Employers use education as a predictor of a worker's level of unobserved cognitive abilities. Because education acts as an imperfect predictor of one's cognitive abilities, and children of poor SES have disadvantages of the type mentioned above in acquiring education, the individual investment in education in the economy distorts productive efficiency, lowers social mobility and increases earnings inequality. The paper will address these issues in a signaling model, adapting the asymmetric information frameworks of Stiglitz (1975) and A. M. Spence (1974).

The rest of the paper is organized as follows: Section 2 describes the basic model of human capital acquisition in a signaling equilibrium framework. Section 4 studies the properties of signaling equilibria specializing to the log-normal case and shows that the use of education as a signal accentuates earnings inequality and reduces social mobility. Section 5 specializes the model to two schooling levels and two levels of unobserved cognitive ability and then studies the nature of equilibrium earnings inequality, social mobility and growth when children of poor SES are adversely affected in obtaining education.

2 The Basic Model

The economy consists of an overlapping generations of agents and of risk neutral competitive producers. In each period there is a continuum of adult population, who live for one period. At the end of the period, he dies and a new adult child is born to each parent. Denote by τ an individual's cognitive ability which affects his productivity at workplace and learning in school. I assume for simplicity that τ is one dimensional, and it takes a $\mathscr{T} = (0, \infty)$. In the set $\mathscr{T}$, a higher number denotes a greater level of cognitive ability. An individual's productivity depends on his schooling level and his level of cognitive ability. The cost of schooling depends on his own schooling level, level of cognitive ability and his family background, denoted here with his parent's schooling level s_{t-1}. The level of schooling can be used to signal one's productivity level. An individual chooses a schooling level s_t which which together with his cognitive ability determine his productivity level, *productivity function* $e(s,\tau)$. Possible education levels are assumed to be from the set $\mathscr{S}=(0,\infty)$, a higher number representing a higher education level.[1] I assume that the cognitive level $\tau_t\in\mathscr{T}$ of a child born to a parent of cognitive ability τ_{t-1} and schooling level s_{t-1} follows probability distribution independent of $\tau_{t-1}$and s_{t-1}, which is characterized by the pdf $g(\tau_t)$[2]

I consider only human capital investment in education, other important forms of human capital investment such as health and nutrition are not considered here. Attainment of an education level by an individual is a more complex decision making process than assumed here. Generally, parents make the initial investments such as preschool investments and investments up to college or so, until the child reaches enough maturity to make his own schooling decision. Family background can have great influence on educational attainment in several other ways. For instance, suppose that the quality of preschool investment of parents' time at home affect children's motivation and persistence to continue schooling. Then, of course, more highly educated parents can provide better learning environment for their children at home. Similarly, more highly educated parents with their better knowledge base of child care, or simply because of their higher incomes can provide better prenatal and post-natal care, and health care for proper cognitive and affective developments of their children.[3]

An individual in period t of ability τ_t and family background $s_{t-1}$is denoted by the index $\xi_t=(\tau_t,s_{t-1})$.The distributions of individuals in period t is characterized by the pdf $f_{\xi_t}(\tau_t,s_{t-1})=g(\tau_t).\pi_{t-1}(s_{t-1})$, where $\pi_{t-1}(.)$ is the pdf of the schooling distribution of the parents in period t.

[1]The general practice in the human capital literature is, however, to treat S as continuous variable, more realistically it is a discrete set.

[2]There is a long controversy over the issue of whether children's innate ability is genetically inherited from parent's innate ability. The scientific consensus is that the correlation between parent's innate ability and a child's innate ability is somewhere between 0.3 to 0.7. I assumed it to be zero, for simplification. There are other controversies regarding talent, ability and intelligence. Some believe that one is born with a fixed level of intelligence, and training and environment has no effect on intelligence. Others do not agree with it, and believe that ability, intelligence and talent could be improved to some extent with better environment and training. Some believe that intelligence or innate ability is fixed when one is born, and less intelligent people can learn and do complex things that we face in our everyday life, in school curricula, and in modern jobs, except that they might take longer, and thus less productive; this is the view we take in this paper.

[3]There are other ways education of parents can influence the educational achievement of their children, for instance, by providing role models.

The effect of the above types of family ground is assumed to affect his cost of obtaining a level of education s_t. Denote this cost function for agent (τ_t, s_{t-1}) by $\theta_t(s_t, \tau_t, s_{t-1})$[4]

Assume that all individuals have identical linear[5] utility function $u(c_t)$, where c_t is the consumption of an adult of period t. An adult of period t with cognitive ability $\tau_t \in \mathscr{T}$ and parental educational background s_{t-1}, takes the wage function $w_t(s_t)$ of period t as given and decides his education level $s_t \in \mathscr{S}$ by solving the following problem:

$$\sigma_t\left(\tau_t, s_{t-1}\right) = \arg\max_{s_t \in \mathscr{S}} u\left(w_t(s_t) - \theta(s_t, \tau_t, s_{t-1})\right) \tag{1}$$

For regular cases, there is a unique optimal solution s_t for each agent $\xi_t = (\tau_t, s_{t-1})$. Notice that in this framework, all individuals with cognitive ability τ_t and family background s_{t-1} behave identically. Denote the optimal solution of the choice problem in Equation 1 for agent (τ_t, s_{t-1}) by $\sigma_t\left(\tau_t, s_{t-1}\right)$.

I assume that the production sector is competitive; the producer is risk neutral; there is no affirmative action in hiring, i.e., workers with the same level of schooling are treated the same way, no matter what their family backgrounds are. In each period $t \geq 1$, a producer announces a wage schedule $w_t(s_t)$ for hiring purposes. He observes the education level s_t of a worker but not his innate productive ability level τ_t. The employer holds a subjective belief about his productivity level $e\left(s_t, \tau_t\right)$ given his education level s_t. This belief is represented by a condition density function $q_t\left(e_t|s_t\right), e \in \mathscr{E}, s_t \in \mathscr{S}$. Perfect competition, and expected profit maximization imply that $w_t(s_t) = \int e_t q_t\left(e_t|s_t\right) de_t$ in equilibrium.

The economy begins at time $t = 1$ with an adult population whose parents' education level is distributed as $\pi_0\left(s\right)$. Given π_0, the transition probability density function $p_t\left(s_t|s_{t-1}\right)$ determines the dynamics of the schooling distributions $\pi_t, t \geq 1$.

The signaling equilibrium is recursively defined over time as follows: At the beginning of time period t, the population density function $\pi_{t-1}\left(s_{t-1}\right)$ is known. A competitive producer knows these and he knows the distribution of τ_t in the population, but he does not observe an individual agent's cognitive ability level τ_t. The employer holds a subjective belief $q(e_t|s_t)$ and announces an earning function $w_t(s_t) = \int e_t q(e_t|s_t) de_t$ for hiring purpose. Given $w_t(s_t)$, each worker (τ_t, s_{t-1})

[4]The assumption that $\theta_t(s_t, \tau_t, s_{t-1})$ varies with τ_t is necessary for education to act as a signal for talent, for justification, see Stiglitz (1975), M. Spence (1973) or Kreps (1990).

[5]Thus we abstract away from bearings on our results from risk sharing between employers and workers.

decides his optimal education level $\sigma_t(\tau_t, s_{t-1})$ as in Equation 1. Given the probability distributions over family backgrounds, $\pi_{t-1}(s_{t-1})$ and innate ability $g(\tau_t)$, the optimal schooling decision variable $\sigma_t(\tau_t, s_{t-1})$ induces a joint probability distribution of (s_t, τ_t) in period t. Denote the joint pdf of the (s_t, τ_t) by $f_{s_t,\tau_t}(.,.)$. This joint distribution of (s_t, τ_t) induces a conditional distribution of τ_t given s_t, denoted by $f_{\tau_t|s_t}(.)$. This conditional distribution together with the productivity function $e_t = e(s_t, \tau_t)$ produces the observed distribution of productivity levels $\hat{q}_t(e|s_t)$ for each level of s_t. We have a signaling equilibrium, when the anticipated distribution coincides with the above observed distribution, i.e., $q_t(e_t|s_t) = \hat{q}_t(e_t|s_t)$ for all education levels that are chosen by some agent in the population.

Notice that optimal schooling choices $s_t = \sigma(\tau_t, s_{t-1})$ determines the transition probability measure $P_t(s_t \in A|s_{t-1})$ of an individual born in the family background s_{t-1} moves to a family background $s_t \in A$ as follows:

$$P_t(s_t \in A s_{t-1}) = \int \mathscr{I}_A(\sigma(\tau_t, s_{t-1}))\, g(\tau_t) d\tau_t \tag{2}$$

The transition probabilities for earnings between period $t-1$ and period t can be defined similarly. The transition probability distribution $P_t(s_t \in A|s_{t-1})$ determines π_t, the distribution of s_t in each period t as follows

$$\pi_t(s_t \in A) = \int P_t(s_t \in A s_{t-1}) d\pi_{t-1}(s_{t-1}). \tag{3}$$

The economy moves to the next period with known π_t and the above process starts all over again.

Initial distribution π^0 of social groups in $\mathcal{S}$, is given. A **signaling equilibrium** is a sequence of probability distributions $\{q_t(e_t|s_t)\}_1^\infty$ and a sequence of optimal schooling decision rules $\{\sigma_t(\tau_t, s_{t-1})\}_1^\infty$ defined in Equation 1 such that at each period $t \geq 1$,

1. The induced wage schedule $w_t(s_t) = \int e_t q_t(e_t|s_t)\, de_t$ is a smooth concave function.

2. Given $w_t(s_t)$, the function $\sigma_t(\tau_t, s_{t-1})$ solves the schooling decision problem in Equation 1 of each agent (τ_t, s_{t-1}).

3. The induced conditional distribution $\hat{q}_t(e_t|s_t)$ of e_t given the optimal solution $s_t = \sigma_t(\tau_t, s_{t-1})$ obtained by using Bayes rule coincides with the anticipated conditional distribution $q_t(e_t|s_t)$ for all s_t.

Assume that

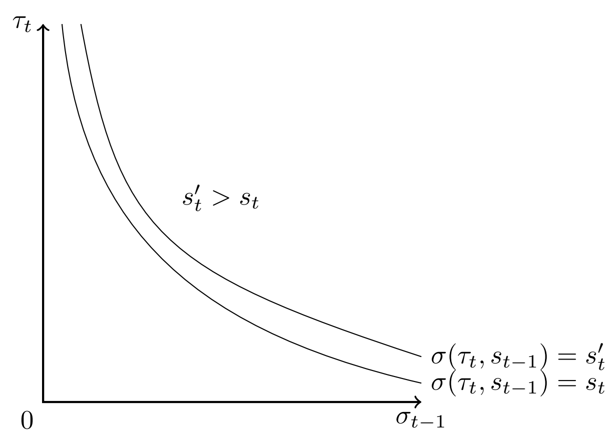
$$\theta\left(s_t, \tau_t, s_{t-1}\right) = \theta_1(s_t).\theta_2(\tau_t, s_{t-1}) \tag{4}$$

where $\theta_1\left(s_t\right)$ is a monotonically increasing smooth function of s_t, and $\theta_2(\tau_t, s_{t-1})$ is a smooth function decreasing in each variable τ_t and s_{t-1}. The distributions of agents in period t is characterized by the pdf $f_{\xi_t}\left(\tau_t, s_{t-1}\right) = g(\tau_t).\pi_{t-1}\left(s_{t-1}\right)$. Assume that there is no affirmative action in hiring, i.e., workers with the same level of schooling are treated the same way, no matter what their family backgrounds are. The first order condition of the schooling choice problem in Equation 1 is given by

$$\frac{w_t'\left(s_t\right)}{\theta_1'\left(s_t\right)} = \theta_2\left(\tau_t, s_{t-1}\right) \tag{5}$$

The left hand side of Equation 5 is a strictly monotonic function of s_t, and hence we can solve s_t as a function of agent characteristics (τ_t, s_{t-1}), which we denote by $s_t = \sigma_t\left(\tau_t, s_{t-1}\right)$.

Figure 1: Sets of individuals (τ_t, s_{t-1}) for whom $\sigma(\tau_t, s_{t-1}) = s_t$ and $\sigma(\tau_t, s_{t-1}) = s_t'$



Note that for each τ_t, s_t, we can solve s_{t-1} as a function of (s_t, τ_t), which we denote by $s_{t-1} = s_t^{*-1}\left(\tau_t, w_t'\left(s_t\right)\theta_1'(s_t)\right)$. Let the bivariate random variable $X_t \equiv (s_t, \tau_t)$ be the optimal schooling level s_t and the cognitive productivity level τ_t of the child $\xi_t = (\tau_t, s_{t-1})$. From the probability distribution of $\xi_t = (\tau_t, s_{t-1})$, we derive the joint probability distribution $f_{(s_t,\tau_t)}\left(s_t, \tau_t\right)$ of s_t, τ_t using the transformation $s_{t-1} = s_t^{*-1}\left(\tau_t, w_t'\left(s_t\right)/\theta_1'(s_t)\right)$, $\tau_t = \tau_t$. Note that the Jacobian of the transformation is given by

$$\frac{\partial(\tau_t, s_{t-1})}{\partial(s_t, \tau_t)} = \det\begin{pmatrix} \frac{\partial \tau_t}{\partial s_t} & \frac{\partial \tau_t}{\partial \tau_t} \\ \frac{\partial s_{t-1}}{\partial s_t} & \frac{\partial s_{t-1}}{\partial \tau_t} \end{pmatrix}$$
$$= -\frac{w_t''(s_t)\,\theta_1'(s_t) - w_t'(s_t)\,\theta_1''(s_t)}{\left[\theta_1'(s_t)\right]^2 \partial\theta_2\left(\tau_t, s_t^{*-1}\left(\tau_t, w_t'(s_t)/\theta_1'(s_t)\right)\right)/\partial s_{t-1}}$$

Hence the joint pdf of s_t, τ_t is given by

$$f_{(s_t,\tau_t)}(s_t, \tau_t) = g(\tau_t)\,.\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left|\frac{\partial(\tau_t, s_{t-1})}{\partial(s_t, \tau_t)}\right|$$

A period-t signaling equilibrium is a wage schedule $w_t(s_t)$ such that

$$w_t(s_t) = \int_T e(s_t, \tau_t)\, f_{\tau_t s_t}(\tau_t)\, d\tau_t = \frac{\int_T e(s_t, \tau_t)\, f_{X_t}(s_t, \tau_t)\, d\tau_t}{\int_T f_{X_t}(s_t, \tau_t)\, d\tau_t}$$
$$= \frac{\int_T e(s_t, \tau_t)\, g(\tau_t)\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left[\frac{\partial\theta_2}{\partial s_{t-1}}\left(\tau_t, s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\right]^{-1} d\tau_t}{\int_T g(\tau_t)\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left[\frac{\partial\theta_2}{\partial s_{t-1}}\left(\tau_t, s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\right]^{-1} d\tau_t}$$
$$= \Psi\left(w_t'(s_t), s_t\right) \text{ say}$$

The right hand side of the above equation is the observed or realized average productivity of the signal class s_t, which in other words, is the conditional expectation of $e(s_t, \tau_t)$ with respect to the observed empirical conditional distribution of τ_t given s_t. The above is a first order non-linear differential equation which under general conditions have smooth solution $w_t(s_t)$, which is unique when we provide an initial condition. We take the initial condition $w(0) = 0$, i.e., the labor with no education has zero productivity.

I do not examine conditions under which there exists a signaling equilibrium. Instead I specialize to log-normal specification of the distributions of productivity level τ_t and the family background level $s_{t-1}$and explicitly compute the signaling equilibrium and study the properties of equilibrium earnings inequality and social mobility.

3 Existence of Equilibrium

I assume the following:

Assumption A1: $\theta_t(s_t, \tau_t, s_{t-1}) = \theta_1(s_t) \cdot \theta_2(\tau_t) \cdot \theta_3(s_{t-1})$, $\theta_1()$ is smooth, monotonically increasing and concave, $\theta_2()$ and $\theta_3(.)$ are smooth, monotonically decreasing.

Assumption A2: The distributions $g(\tau)$ and $\pi_0(s_0)$ belong to a concave conjugate family.

Theorem 3.1. *Under Assumption A1 and Assumption A2, there exists a signalling equilibrium.*

Proof. Suppose we have found a smooth concave wage schedule $w_t(s)$ with a first derivative $w_t'()$. The first order condition of the optimization problem in Equation 6 is given by

$$\frac{w_t'(s_t)}{\theta_1'(s_t)} = \theta_2(\tau_t)\,\theta_3(s_{t-1}) \tag{6}$$

Since by Assumption A1, the left hand side of Equation 6 is a monotonic function of s_t, one can uniquely solve s_t as a function of agent characteristics (τ_t, s_{t-1}), which produces the optimal schooling decision rule, $\sigma_t(s_t, \tau_t)$ in the definition of signaling equilibrium. For each τ_t, s_t, one can find a unique s_{t-1} from Equation 6. For given τ_t, consider the 1-1 and onto transformation $s_t \longmapsto s_{t-1}$ defined by $s_{t-1} = \phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)$. The Jacobian of the transformation is given by

$$\frac{ds_{t-1}}{ds_t} = -\frac{w_t''(s_t)\,\theta_1'(s_t) - w_t'(s_t)\,\theta_1''(s_t)}{\left[\theta_1'(s_t)\right]^2 \theta_3'\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)}$$

Hence the pdf of X_t is given by

$$f_{s_t,\tau_t}(s_t, \tau_t) = g(\tau_t)\,.\pi_{t-1}\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)\left|\frac{ds_{t-1}}{ds_t}\right|$$

From the above, the conditional distribution of τ_t given s_t is given by

$$\begin{aligned}
f_{\tau_t s_t}(\tau_t s_t; w_t'(s_t)) &= \frac{f_{s_t,\tau_t}(s_t, \tau_t)}{\int f_{s_t,\tau_t}(s_t, \tau_t)\,d\tau_t} \\
&= \frac{g(\tau_t)\,.\pi_{t-1}\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)\cdot\theta_3'\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)}{\int g(\tau_t)\,.\pi_{t-1}\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)\cdot\theta_3'\left(\phi_t\left(\frac{w_t'(s_t)}{\theta_1'(s_t)\theta_2(\tau_t)}\right)\right)d\tau_t}
\end{aligned}$$

In equilibrium we should have

$$\begin{aligned}
w_t(s_t) &= \int e(s_t, \tau_t)\,f_{\tau_t s_t}(\tau_t s_t; w_t'(s_t))\,d\tau_t \\
&\equiv \Psi(s_t, w_t'(s_t)) \text{ say}
\end{aligned}$$

The above is a non-linear first order differential equation. The existence of a signaling equilibrium boils down to the question, does the above differential equation has a solution $w_t(s)$ which is smooth and concave and satisfying the condition $w_t(0) = 0$.

How many?

I show now that the above has a unique solution.

The proof follows the steps in the proof of Theorem 2 in Quinzii and Rochet (1985).

■

Q.E.D.

4 Log-Normal Economy

I examine how inequality in earnings and schooling changes over time with the specification of log-normal distributions for cognitive ability τ and the initial distribution of population over the observed schooling levels of parents.

The notation $X \sim \Lambda\left(\mu, \sigma^2\right)$ means the random variable X is log-normally distribution with parameters μ, and σ^2, i.e. $\ln X$ is normally distributed with mean μ and variance σ^2. Assume that

$$\begin{aligned} s_{t-1} \sim & \quad \Lambda\left(\mu_{s_{t-1}}, \sigma^2_{s_{t-1}}\right) \\ \tau_t \sim & \quad \Lambda\left(\mu_\tau, \sigma^2_\tau\right) \\ e(s_t, \tau_t) = s_t^{\rho}.\tau_t, \ \rho > 0. & \end{aligned}$$

For simplicity and without loss of much generality, I assume that $\theta_1(s_t) = s_t$. I consider two cases below. First I consider the case in which cost of education depends only on τ and not on family background s_{t-1}. In this case, from the observable optimal schooling level s_t, the unobserved cognitive ability level τ_t or the productivity level $e(s_t, \tau_t)$ of the worker could be predicted perfectly. Then I consider the case in which cost of education depends on both τ_t and the family background s_{t-1}. In this case the observed optimal education level can predict the unobserved cognitive skill level imperfectly. I then compare how the distribution of earnings and education levels become more unequal due to signaling role of education.

4.1 Signaling cost does not depend on family background

Assume that cost of education does not depend on the family background of the child, i.e. family background does not have effect on child development. Let it be more specific as $\theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}$ where, $\alpha > 0$. The pdf of τ_t is assumed to be log-normal as follows:

$$f(\tau_t) = \frac{1}{(2\pi\sigma_\tau)^{1/2}\tau}\exp\left\{-\frac{1}{2}\left[\frac{(\ln\tau - \mu_\tau)^2}{\sigma_\tau^2}\right]\right\}$$

From the first order condition of the agent's schooling choice problem Equation 5, we have $w_t'(s_t) = \tau_t^{-\alpha}$. Note that given schooling level s_t, one can perfectly predict his ability level τ_t as I mentioned earlier. Denote this forecasting rule by $\tau_t = (w_t'(s_t))^{-1/\alpha}$. This prediction of τ_t given the optimal schooling level s_t is equivalent to the degenerate pdf $f_{\tau_t|s_t}(\tau_t|s_t) = 1$ if $\tau_t = (w_t'(s_t))^{-1/\alpha}$ and $f_{\tau_t|s_t}(\tau_t|s_t) = 0$ otherwise. The equilibrium is attained if $w_t(s_t) = \int e(s_t, \tau_t) f_{\tau_t|s_t}(\tau_t|s_t)\, d\tau_t$, which in our case simplifies to the following first order differential equation,

$$\frac{dw_t(s_t)}{ds_t} = \left[\frac{s_t^\rho}{w_t(s)}\right]^\alpha$$

The above first order non-linear differential equation is in the separation-of-variables form and can be solved explicitly. The general solution of this differential equation is given by

$$w_t(s_t) = \left[c + \frac{\alpha+1}{\alpha\rho+1} s_t^{\alpha\rho+1}\right]^{\frac{1}{1+\alpha}}, \text{ where } c \text{ is a constant of integration.}$$

Each value of c will give a signaling equilibrium and there are continuum of them. Using the initial condition $w(0) = 0$, the equilibrium earnings function becomes,

$$w_t(s_t) = \left[\frac{\alpha+1}{\alpha\rho+1}\right]^{\frac{1}{1+\alpha}} s_t^{\frac{\alpha\rho+1}{\alpha+1}}. \tag{7}$$

To compute the equilibrium distribution of schooling levels and the wages, note that $\tau_t = (w_t'(s_t))^{-1/\alpha}$. Substituting the value of $w_t'(s)$ from the above, and taking natural log, i.e. ln on both sides, we see that $\ln(s_t) = \frac{1}{1-\rho}\ln\frac{\alpha\rho+1}{1+\alpha} + \frac{\alpha+1}{1-\rho}\ln\tau$. Thus, the equilibrium schooling distribution in period t follow the following log-normal distribution,

$$s_t \sim \Lambda\left(\frac{1}{1-\rho}\ln\frac{\alpha\rho+1}{1+\alpha} + \frac{\alpha+1}{1-\rho}\mu_\tau, \left[\frac{1+1/\alpha}{1-\rho}\right]^2 \alpha^2\sigma_\tau^2\right) \tag{8}$$

and the equilibrium wage distribution in period t is also a log-normal,

$$w_t \sim \Lambda\left(\frac{\rho}{1-\rho}\ln\frac{\alpha\rho+1}{\alpha+1}+\frac{\alpha\rho+1}{1-\rho}\mu_\tau,\left[\frac{\alpha\rho+1}{\alpha+1}\cdot\frac{1+\rho/\alpha}{1-\rho}\right]^2\alpha^2\sigma_\tau^2\right). \tag{9}$$

One can compute the transition probability density function for schooling levels of two generations $f_{s_t|s_{t-1}}(s_t|s_{t-1})$ and the transition probability density function for earnings $f_{w_t|w_{t-1}}(w_t|w_{t-1})$ and compute a measure of schooling mobility and earnings mobility.

The Gini-coefficient for schooling inequality in period t is

$$G_{s_t} = 2\Phi\left(\frac{1}{\sqrt{2}}\frac{1+1/\alpha}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2}\right)-1$$

$$G_{w_t} = 2\Phi\left(\frac{1}{\sqrt{2}}\frac{\alpha\rho+1}{\alpha+1}\cdot\frac{1+1/\alpha}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2}\right)-1$$

where Φ is the $erf(x)$ function. The Gini coefficient of earning distribution is smaller than the Gini coefficient of schooling distribution. How these compare when schooling cost depends on the family background, and thus education is an imperfect signal of cognitive skill.

4.2 Signal cost depends on ability and family background

I now assume that schooling cost depends on family background, which is incorporated by assuming that $\theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}\cdot s_{t-1}^{-\gamma}$, where, $\alpha,\gamma > 0$. I assume that family background of is log-normally distributed as $s_{t-1}w_t \sim \Lambda\left(\mu_{s_t-1},\sigma^2_{s_{t-1}}\right)$. The rest of the specifications are as in the previous subsection.

The joint pdf of (τ_t, s_{t-1}) is given by

$$f_{(\tau_t,s_{t-1})}(\tau_t,s_{t-1}) = \frac{1}{2\pi\sigma_\tau\sigma_{s_{t-1}}\tau_t s_{t-1}}\exp\left\{-\frac{1}{2}\left[\frac{(\ln\tau_t-\mu_\tau)^2}{\sigma_\tau^2}+\frac{\left(\ln s_{t-1}-\mu_{s_{t-1}}\right)^2}{\sigma^2_{s_{t-1}}}\right]\right\}$$

The first order condition of the agent is

$$w_t'(s_t) = \theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}\cdot s_{t-1}^{-\gamma}$$

Assume as before $\theta_t'(s_t) = 1$. Notice that the above implicitly defines a transformation

$(\tau_t, s_{t-1}) \longmapsto (\tau_t, s_t)$ and the Jacobian of this transformation is given by

$$\begin{aligned}\frac{\partial(\tau_t, s_{t-1})}{\partial(\tau_t, s_t)} &= \det\begin{bmatrix} 1 & 0 \\ \frac{\partial s_{t-1}}{\partial \tau_t} & -\frac{1}{\gamma}\left[w_t'(s_t)\right]^{-\frac{\gamma+1}{\gamma}} \cdot \tau_t^{-\frac{\alpha}{\gamma}} \cdot w_t''(s_t) \end{bmatrix} \\ &= -\frac{1}{\gamma}\left[w_t'(s_t)\right]^{-\frac{\gamma+1}{\gamma}} \cdot \tau_t^{-\frac{\alpha}{\gamma}} \cdot w_t''(s_t)\end{aligned}$$

Thus the joint pdf of (s_t, τ_t) is given by

$$f_{(s_t,\tau_t)}(s_t, \tau_t) = \frac{w_t''(s_t)}{2\pi\sigma_\tau\sigma_{s_{t-1}}\tau_t w_t'(s_t)} e^{-\frac{1}{2}\left[\frac{1}{\sigma_\tau^2}(\ln\tau_t - \mu_\tau)^2 + \frac{1}{\sigma_{s_{t-1}}^2}\left(\frac{\alpha}{\gamma}\ln\tau_t + \frac{1}{\gamma}\ln w_t'(s_t) + \mu_{s_{t-1}}\right)^2\right]}$$

The bracketed term in the above exponential can be rewritten as

$$\begin{aligned}[\cdot] &= \frac{1}{\sigma_\tau^2}(\ln\tau_t - \mu_\tau)^2 + \frac{1}{\sigma_{s_{t-1}}^2}\left(\frac{\alpha}{\gamma}(\ln\tau_t - \mu_\tau) + \left[\frac{1}{\gamma}\ln w_t'(s_t) + \left(\mu_{s_{t-1}} + \frac{\alpha}{\gamma}\mu_\tau\right)\right]\right)^2 \\ &= \left(\frac{1}{\sigma_\tau^2} + \frac{\alpha^2}{\gamma^2\sigma_{s_{t-1}}^2}\right)(\ln\tau_t - \mu_\tau)^2 + 2\frac{\alpha}{\gamma^2\sigma_{s_{t-1}}^2}(\ln\tau_t - \mu_\tau)\cdot\left[\ln w_t'(s_t) + \left(\gamma\mu_{s_{t-1}} + \alpha\mu_\tau\right)\right] \\ &\quad + \text{ a term involving } w_t'(s_t) \text{ but not } \tau_t \\ &= \frac{\left[\ln\tau_t - \left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right)\right]^2}{\sigma^{*2}} \\ &\quad + \text{ a term involving } w_t'(s_t) \text{ but not } \tau_t\end{aligned}$$

where

$$\beta^* = \frac{\alpha\sigma_\tau^2}{\gamma^2\sigma_{s_{t-1}}^2 + \alpha^2\sigma_\tau^2}$$

and

$$\sigma^{*2} = \frac{\gamma^2\sigma_\tau^2\sigma_{s_{t-1}}^2}{\gamma^2\sigma_{s_{t-1}}^2 + \alpha^2\sigma_\tau^2}$$

Hence the conditional pdf of $\tau_t|s_t$ is given by

$$f_{\tau_t s_t}(\tau_t) = \frac{1}{\sqrt{2\pi}\sigma^*\tau_t}\exp\left\{-\frac{1}{2}\frac{\left[\ln\tau_t - \left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right)\right]^2}{\sigma^{*2}}\right\}$$

which is a log-normal distribution.

In this case, we have

$$\begin{aligned}w_t(s_t) &= \int e(s_t, \tau_t)\, f_{\tau_t s_t}(\tau_t)\, d\tau_t \\ &= s_t^\rho . \exp\left\{\left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right) + \sigma^{*2}/2\right\} \\ &= s_t^\rho \exp\{d - \beta^*\ln w_t'(s_t)\}, \text{ where } d = [1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} + \sigma^{*2}/2\end{aligned}$$

from which we have

$w_t'(s_t) = \left[\frac{s_t^{\rho}\tilde{\mu}}{w_t(s_t)}\right]^{1/\beta^*}$, where $\tilde{\mu} = \exp(d) = \exp\left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} + \sigma^{*2}/2\right)$

A general solution of this differential equation is given by

$$w_t(s_t) = \left[c + \frac{1+\beta^*}{\rho+\beta^*}\tilde{\mu}^{1/\beta^*} s_t^{(\rho+\beta^*)/\beta^*}\right]^{\beta^*/(1+\beta^*)}$$

where, c is a constant of integration. The above is a one parameter family, each c represents a signaling equilibrium with an associated self-fulfilling employer expectations regarding the relationship between education level and productivity level.[6] Using the same initial condition $w_t(0) = 0$ as in the previous subsection, we have $c = 0$. Thus, equilibrium wage function is given by,

$$w_t(s_t) = \tilde{\mu}^{1/(1+\beta^*)}\left[\frac{1+\beta^*}{\rho+\beta^*}\right]^{\beta^*/(1+\beta^*)} s_t^{(\rho+\beta^*)/(1+\beta^*)} \tag{10}$$

We want to find the equilibrium income distribution, i.e., the distribution of s_t, and invariant distribution for (s_t, τ_t) and the long-run growth rate.

To find the equilibrium distribution of s_t, let us denote by $z = w_t'(s_t)$. Notice that $z = \tau_t^{-\alpha} \cdot s_{t-1}^{-\gamma}$. Thus we know that $z \sim \Lambda\left(-\alpha\mu_\tau - \gamma\mu_{s_{t-1}}, \alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right)$. Under the assumption that $c = 0$, we have

$$w_t'(s_t) = K \cdot s_t^{(\rho-1)/(1+\beta^*)}, \text{ where } K = \left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)^{1/(1+\beta^*)} \cdot \tilde{\mu}^{1/(1+\beta^*)}$$

Hence, we have $\ln s_t = \frac{\ln\left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)}{1-\rho} + \frac{(1+\beta^*)}{1-\rho}(\alpha\ln\tau + \gamma\ln s_{t-1})$. Hence we have that

$$s_t \sim \Lambda\left(\left[\frac{\ln\left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)}{1-\rho} + \frac{(1+\beta^*)}{1-\rho}\left(\alpha\mu_\tau + \gamma\mu_{s_{t-1}}\right)\right], \frac{(1+\beta^*)^2}{(\rho-1)^2}\cdot\left[\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right]\right) \tag{11}$$

and

$$w_t s \sim \Lambda\left(\mu_w, \left[\frac{\rho+\beta^*}{1+\beta^*}\cdot\frac{(1+\beta^*)}{1-\rho}\right]^2\left[\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right]\right) \tag{12}$$

For this economy, the Gini-coefficient for schooling inequality in period t is

[6] We should check what happens to net income for each agent $\xi_t = (\tau_t, s_{t-1})$ as c changes, and check to see if $c = 0$, gives the highest net income.

$$G'_{s_t} = 2\Phi\left(\frac{1}{\sqrt{2}}\frac{1+\beta^*}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2}\right) - 1.$$

$$G'_{w_t} = 2\Phi\left(\frac{1}{\sqrt{2}}\frac{\rho+\beta^*}{1+\beta^*}\cdot\frac{1+\beta^*}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2}\right) - 1.$$

Comparing Gini coefficients for schooling level, $G'_{s_t} < G_{s_t}$ and $G'_{w_t} < G_{w_t}$. Compared to the previous case, the Gini-coefficient has two sources of variation variance of τ_t and σ_{t-1}. However, it is clear from Figure 1 that the signaling equilibrium in the second case has pooling of individuals of varying cognitive skills from various family backgrounds who chose same schooling level and thus earned the same wage. This will make the schooling inequality and earnings inequality smaller in the second case.

It is possible that when other labor market mechanisms such as quits layoffs which would break the pooling of individuals to smaller and finer subclasses and thus the inequality will be further increased and social mobility will also improve. We see in the log-normal case that when labor market mechanisms that help to lower the pooling groups in the equilibrium increases inequality and increases social mobility. I show those in the finite case in the next section.

5 Finite number of ability and schooling types

To gain further insights about the nature of the equilibrium dynamics of earnings inequality, social mobility and growth, I consider the following simple economy for much of this paper. Let $\mathscr{T} = \{1, 2\}$, $S = \{1, 2\}$. Assume that the number of talented workers with high education level create social productive knowledge which generate growth in earnings.

$$e(s,\tau) = \begin{cases} e_1 & \text{if } s = 1, \forall \tau \in \mathscr{T} \\ e_2 & \text{if } s = 2, \tau = 1 \\ e_3 & \text{if } s = 2, \tau = 2 \end{cases} \tag{13}$$

An interpretation of the above is that the workers with education level 1 are unskilled workers and the talent of the unskilled workers do not affect their productivity; however, higher educated talented workers have higher productivity than higher educated not-so-talented workers.

Does there exist any signaling equilibrium, and if there exists one, are there many equilibria? Is there an equal opportunity separating equilibrium? Does any of these equilibria attain maximal growth and social mobility? The answers to these questions depend on the productivity technology $e(\tau, s)$ and the cost function, $\theta(s_t, \tau_t, s_{t-1})$. I assume that the cost function $\theta(s_t, \tau_t . s_{t-1})$ satisfies the following:

$$\left.\begin{array}{c}\theta(1, \tau_t, s_{t-1}) = 0 \forall \tau_t, s_{t-1}, \text{ and} \\ \theta(2,2,2) < \theta(2,1,2) < (e_2 - e_1) + p\,(e_3 - e_2) < \theta(2,2,1) < \theta(2,1,1)\end{array}\right] \tag{14}$$

Signaling equilibrium 1: Suppose the employers in period t hold the following subjective probability distribution $q_t\,(e|s)$)of productivity level e given his schooling level s, which in matrix form is given by

$$[q_t\,(es)]_{\substack{e=e_1,e_2,e_3 \\ s=1,2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1-p \\ 0 & p \end{bmatrix}$$

Given the above expectations, the employer announces the following wage schedule:

$$w_t(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1-p) + e_3.p & \text{if } s_t = 2 \end{cases} \quad \text{for all } t \geq 0$$

Given the above wage schedule, one can easily verify that the equilibrium schooling decisions $\sigma_t(\tau_t, s_{t-1})$ of an agent of talent type τ_t from the family background s_{t-1} is as follows:

$$\sigma_t(\tau_t, s_{t-1}) = \begin{cases} 1 & \forall \tau_t \in \mathcal{T} if s_{t-1} = 1 \\ 2 & \forall \tau_t \in \mathcal{T} if s_{t-1} = 2 \end{cases} \quad \text{for all } t \geq 0$$

It can be easily checked that given the above optimum solution, the observed conditional probability distribution of e given s_t will coincide with the anticipated one. Note that the the transition matrix associated with $\sigma_t(.)$ is the following:

$$P_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \forall t \geq 0$$

Thus in this economy there is no intergenerational mobility. Furthermore, the economy is in steady-state from the beginning. Thus, the number of highly talented highly educated workers $R_t = p \cdot \pi_0^2$, and hence the productivity growth rate is given by $\gamma(p\pi_0^2)$ which is strictly less than $\gamma(p)$, the maximum attainable productivity growth rate for the economy when all talented individuals from all socio-occupational groups obtain higher education.

This equilibrium is not equal opportunity separating, nor maximal growth separating type. In this equilibrium, all talent types of the children from each type of family backgrounds are pooled.

Could there be any other equilibrium for the above economy? For a certain subclass of the above economies, there is another equilibrium, which is growth enhancing separating and is Pareto superior to the above equilibrium. To see this, consider the following:

Signaling equilibrium 2: let $v_t \equiv \frac{p}{p\pi^1_{t-1}+\pi^2_{t-1}}$. Note that $v_t > p\ \forall t \geq 1$. At $t=1$, v_1 is known. Let us suppose that apart from the assumption in Equation 14, the cost function also satisfies the condition:

$$\theta(2,2,1) < (e_2 - e_1) + v_1\,(e_3 - e_2) < \theta(2,1,1)$$

Suppose the employer holds the following subjective probability distribution for the productivity type E_t given S_t:

$$\overline{q}_t\,(es) = \begin{bmatrix} 1 & 0 \\ 0 & 1-v_t \\ 0 & v_t \end{bmatrix} \quad \text{for all } t \geq 1 \tag{15}$$

According to Equation 4, given above expectations, the employer announces the following wage schedule:

$$\overline{w}(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1-v_t) + e_3.v_t & \text{if } s_t = 2 \end{cases}$$

Given the above wage schedule, the original $\sigma_t(\tau_t, s_{t-1})$ will be optimal for all (τ_t, s_{t-1}) except for $\tau_t = 2$, $s_{t-1} = 1$, who will choose $s_t = 2$. It can be easily checked that for this optimal solution, the observed conditional probability distribution of $ e_{t}$ given s_t will coincide with the anticipated one in Equation 15. Note that the transition matrix associated with this new optimal schooling decision $\overline{s}^*_t(.)$ is as follows:

$$\overline{P}_t = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}$$

Thus in this economy there is intergenerational mobility. The proportion of population with higher education will go on increasing and the proportion of the population with lower education will go on decreasing. This process, however, cannot go on for ever, since in that case $v_t \to p$, as $t \to \infty$, which will mean that there will be some finite $t_0 > 1$ such that $v_{t_0} > \theta(2,2,1)$ for the first time and then on the equilibrium will switch on to the previous one with no mobility. Note, however that the new steady-state equilibrium growth rate will be $\gamma\left(\pi^2_{t_0} \cdot p\right)$ since $\pi^2_{t_0} > \pi^2_0$. Furthermore, the short-run growth rate up to period t_0, is higher in the second equilibrium than in the first type; and the second equilibrium is Pareto superior to the first.

Furthermore, notice that there will be a positive wage growth during all periods $t \leq t_0$, and after t_0, the source of growth is only from factor productivity growth.

Thus, in this economy there may exist multiple equilibria; which one will actually realize depends on the expectations of the employers. The question is then, how the employer's expectations are formed? We need a theory of expectations formation of the producers to select an equilibrium, and we do not pursue this theory here.

Also note that the first signaling equilibrium will be in stationary state from time $t = 1$, will produce no social mobility in any periods. The second signaling equilibrium will produce upward mobility from social class $s = 1$ to $s = 2$ up to time $t = t_0$ according to the transition matrix $\overline{P}_t$, and during this period, there will be a positive wage growth due to upward mobility; after period t_0, however, the process will revert to the mobility pattern of the first signaling equilibrium. Two equilibria, however, will produce two different long-run income distributions.

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