Article

Family Expansion and Capital Accumulation of a Dynasty

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Abstract

We consider a unified framework that combines two strands of previous literature on overlapping generations growth models of endogenous fertility and savings: one strand incorporating two-period lived agents with life-cycle utility functions and the other strand incorporating one period lived agents with dynastic utility functions. In this framework, we study the long-run effects of unfunded social security on fertility and savings. We provide complete characterization of optimal path in terms of the life-cycle felicity index and the degree of altruism towards all the future offsprings, exhibiting either monotonicity of the standard growth model, fluctuations of the Easterlin (1987) hypothesis or convergence in finite period.

Keywords

Endogenous growth, endogenous fertility, capital accumulation, growth dynamics

Introduction

We consider a unified framework that combines various strands of literature on overlapping generations growth models of endogenous fertility and savings and study the long run effect of unfunded social security on fertility and savings.

Empirical studies on social security of developed countries examine its effect on savings rate. Although most developing countries do not have a formal pay-as-you-go social security programme, quite a few countries have recently introduced social security programmes covering part of their population. Based on data from these countries, a number of studies estimated the effect of social security on the fertility level. While most studies on developed and developing countries found

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controversial effects (see for instance, Barro and Feldstein (1978) for their controversies on the US and see Nugent (1985) for a summary of findings on developing countries), general consensus is that the effect is negative on fertility and savings. Not much is known about how social security jointly affects both fertility and savings rates. Using an Italian time series data set, Cigno and Rosati (1992), however, found that social security has negative effect on fertility and its effect on savings is positive or negative based on whether the social security is fully funded or has a deficit.

A pay-as-you-go social security programme transfers income from younger generations to older generations in each period. Therefore, the overlap of generations is necessary in modelling social security. Modelling of endogenous fertility and savings depends on the motive for savings and having children. Growth models with endogenous fertility and savings specify such motives with mainly two types of utility functions: the altruistic or dynastic utility function and the life-cycle utility function. Almost all models that deal with dynastic utility function assume that each person lives for one period. More specifically, denote the consumption of an adult of generation t by c_t^t and the number of children he has by n_t. Barro and Becker (1989) assume that the utility of an adult of generation t, V_t, is given by

$$V_t = u(c_t^t) + \gamma(n_t)V_{t+1}$$

where $u(c_t^t)$ is the felicity index. They further assume $\gamma(n_t)$ to be Cobb-Douglas type (see Benhabib and Nishimura (1989) for the case of general concave functions). Kemp and Kondo (1986), Lapan and Enders (1990) and Nishimura and Kunaponagkul (1991) all assume that $\gamma(n_t)$ is constant. In such models, to incorporate a motive for children in the utility function, the tradition has been to assume that the felicity index u depends on c_t^t as well as on n_t. Since there is no overlap of active generations, there is no scope for exchanges among the living agents of different generations within a time period. Thus in these models, the sole purpose of saving is the bequest and the motive for having children is to derive utility analogous to deriving utility from consumption of physical commodities.

Growth models of endogenous fertility and savings that use life-cycle utility functions, on the other hand, assume that each person lives for two periods, the adulthood and the old-age. Suppose we denote the consumption of an adult of period t by c_t^t when he is adult and by c_{t+1}^t in the next period when he is old. These models explicitly allow overlap of generations and study exchanges among living generations. Raut (1991, 1992) and Raut and Srinivasan (1994) assume that the felicity index u of an individual depends only on her own consumption c_t^t, and c_{t+1}^t. The motive for saving in these models is old-age pension. To incorporate a motive for having children in these models, they assume that children transfer a fraction of their income to their old parents and this fraction is determined exogenously by social norms. Children in these models are treated as poor man's capital. Eckestein and Wolpin (1985), on the other hand, incorporate a motive for having children by assuming that the felicity index u depends not only on c_t^t and c_{t+1}^t but also on n_t and thus children in their model are treated analogous to consumption good.

There is a third strand of literature in which the agents have life cycle utility with limited altruism in overlapping generations model. Veall (1986), and Nishimura and Zhang (1992) assume that an agent's utility depends not only on her own life time consumption in two periods, but also on her parent's old-age consumption. This allows Nishimura and Zhang to endogenize transfers from children to parents. Raut (1992b) assumes that an agent's utility depends not only on her own life time consumption, but also on her parents' old-age consumption and her children's young-age consumption. Raut thus endogenizes both old-age transfers from children to parents and bequest transfers from parents to children.

To examine the effect of social security on the long-run rates of growth in population, capital accumulation and income, it is appropriate to have overlap of generations structure as in the second and third strands of the literature. In this article we retain the overlap of generations structure of the second and third strand of the literature and combine it with dynastic welfare concern as in the first strand of literature. This provides a unified framework that includes most of the previous models as special cases. In this unified framework, we study the effect of social security on fertility and savings. We characterize dynamics of an optimal path in terms of the properties of the felicity index, u and the degree of altruism towards future generations, $\gamma(\cdot)$. We find conditions which generate fluctuations in the equilibrium path of capital labour ratio, k_t and fertility n_t that are consistent with the Easterlin (1987) hypothesis which states that in time series over a long period of time, the periods when households have higher income, they also have higher fertility and which in turn leads to lower income and fertility level in the next period and repeating this process over time.

In the second section we present our general framework. In the third section, we study effects of social security on steady-state fertility, savings and welfare of generations. In the last and fourth section, we characterize the dynamics of optimal path of our unified model and extend the results that are known for one period lived agent models.

Basic Framework

Production Sector

We assume that the productive sector has a constant returns to scale production function $Y_t = F(K_t, L_t)$ which uses capital K_t and labour L_t to produce output Y_t in each period t, $t \geq 0$. Capital takes one period to gestate. Old members of the households own capital. We adopt the convention that the producer borrows from the old members of the households the stock of capital K_t at the beginning of period t and pays them $(\partial F/\partial K)K_t$ amount of rental income during the period t and stock of depreciated capital $(1-\delta)K_t$. This depreciated capital, $(1-\delta)K_t$, is bequeathed to the L_t children by the L_{t-1} old parents at the end of period t when they die. Thus, at the beginning of period $t+1$ the stock of capital available for production is:

$$K_{t+1} = (1-\delta)K_t + L_t s_t \tag{1}$$

On the right hand side of the above, the first term is the inherited capital and the second term is the new capital added by the adults of period t. We assume that $s_t \geq 0$, which is equivalent to the assumption that capital is irreversible.
From (1) we have the following relationship:

$$k_{t+1} = \frac{(1-\delta)k_t + s_t}{n_t} \tag{2}$$

where n_t is the number of children chosen by an adult of period t.

Households

At the beginning of time, $t = 0$, assume that there is only one adult agent who has at her disposal an initial endowment of capital $k_0 > 0$. Each person lives for three periods: young, adult and old. While young she is dependent on her parents for all decisions including childhood consumption. As an adult, she earns income w_t in the labour market, out of which she pays τw_t amount of social security taxes. Thus, $(1-\tau)w_t$ is her budget during her adulthood. Given her budget, she decides the amount of savings s_t and the number of children $n_t \geq 0$. The child-rearing cost in period t is θ_t per child in the unit of period t income. In the next period, she inherits $(1-\delta)k_t$ amount of physical capital assets from her deceased parents and lives off the income from social security benefits, b_{t+1} and returns from her assets, $\rho_{t+1}[(1-\delta)k_t + s_t]$, where ρ_{t+1} is the rental rate of capital in period $t+1$.

We assume that utility of agent t, V_t depends on her own life cycle consumption and the discounted sum of the utilities of her children V_{t+1} as follows:

$$V_t = u(c_t^t, c_{t+1}^t) + \delta(n_t) n_t V_{t+1} \tag{3}$$

where $\delta(n_t)$ is the weight given to each child's utility. We assume that $\delta(n_t)$ is decreasing function of the number of children, n_t. We denote by $\gamma(n_t) = \delta(n_t)\, n_t$ and assume that $\gamma(n_t) < 1$.

The recursive equation (3) leads to the following welfare for agent $t = 0$ as a function of the stream of lifetime consumption and fertility level of future generations:

$$V_0 = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \gamma(n_s) \right) u(c_t^t, c_{t+1}^t) \tag{4}$$

Assuming perfect foresight and complete enforceability of her decisions $\{n_t, k_{t+1}\}_0^\infty$ on subsequent generations and for given stream of future social security benefits $\{b_{t+1}\}_0^\infty$, the problem of the adult of generation $t = 0$ could be formally stated as follows:

$$
\begin{gathered}
\underset{\{(n_t, k_{t+1})\}_0^\infty}{\text{Maximize}} \qquad V_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\gamma(n_s)\right)u(c_t^t, c_{t+1}^t) \\
\textit{subject to} \\
c_t^t = (1-\tau)w_t - s_t - \theta_t n_t \\
c_{t+1}^t = \rho_{t+1}\left[(1-\delta)k_t + s_t\right] + b_{t+1} \\
k_{t+1} = \tfrac{(1-\delta)k_t + s_t}{n_t} \\
\rho_{t+1} = f'(k_{t+1}) \\
w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \text{ say} \\
k_0 \text{ (and thus) } w_0 \ \textit{given},\ t \ge 0
\end{gathered}
\tag{5}
$$

where the social security benefits and revenues satisfy:

$$b_{t+1} = \tau n_t w_{t+1} \tag{6}$$

An *equilibrium* is a sequence $\{n_t^*, k_{t+1}^*\}_0^\infty$ such that $\{n_t^*, k_{t+1}^*\}_0^\infty$ solves (5) for given sequence of benefits $\{b_{t+1}\}_0^\infty$ in (6) with $n_t w_{t+1}$ replaced by $n_t^* w(k_{t+1}^*)$.

Assume that the utility function, the production function and the degree of altruism are all concave and increasing; furthermore, assume that $\gamma(0) = 0$ and there exists a positive constant $\bar{\gamma} < 1$ such that $\gamma(n) \le \bar{\gamma}$ for all n. Under these conditions, the solution of the above problem in (5) is equivalent to the solution of the following Bellman equation of the dynamic programming problem:

$$
\begin{gathered}
V(k_t) = \underset{(n_t, k_{t+1})}{\textit{maximize}}\,[u(c_t^t,\ c_{t+1}^t) + \gamma(n_t)V(k_{t+1})] \\
\textit{subject to} \\
c_t^t = (1-\tau)w(k_t) + (1-\delta)k_t - (\theta + k_{t+1})n_t \\
c_{t+1}^t = f'(k_{t+1})k_{t+1}n_t + b_{t+1} \\
k_0 > 0 \ \textit{given.}
\end{gathered}
\tag{7}
$$

In equation (7), the $\{n_t, k_{t+1}\}_0^\infty$ is found for exogenously given sequence of benefits $\{b_{t+1}\}$.

Determination of Steady-state and Comparative Statics

A 'non-trivial steady state equilibrium' of the economy is a pair $(k^*, n^*) \in R_{++}^2$ such that in problem (7), $k_t = k^*$ and $b_{t+1} = \tau n^* w(k^*) \Rightarrow$ the optimal $k_{t+1} = k^*$ and the optimal $n_t = n^*$ for all t, $t \ge 0$.

We assume that the value function defined in (7) is concave around a steady state and we restrict our attention to the convex region, D, on which $V(k)$ is differentiable and contains a steady-state in its interior. This restriction is necessary since the value function is not concave in general even when we assume that the production function, the utility function and the degree of altruism function are all concave. As long as an optimal solution path starts from D i.e., $k_0 \in D$ the path is unique.

Let us denote by $R(k) \equiv f(k) - (1 - \tau)w(k)$. Let the steady-state consumption of an agent be denoted by c_1^* when adult and c_2^* when old. Then we have

$$c_1^* \equiv (1-\tau)w(k^*) + (1-\delta)k^* - (\theta + k^*)n^* \tag{8}$$

$$c_2^* \equiv n^* R(k^*) \tag{9}$$

If there exists a steady-state (k^*, n^*), then plugging them in (7), we obtain the following:

$$V(k^*) = \frac{u(c_1^*, c_2^*)}{1-\gamma(n^*)} \tag{10}$$

From the first order necessary conditions for an interior solution in (7), a non-trivial steady-state solution (k^*, n^*) must also satisfy the following conditions:

$$-u_1.(\theta + k^*) + u_2 f'\left(k^*\right)k^* + \gamma'\left(n^*\right)V(k^*) = 0 \tag{11}$$

$$-u_1 n^* + u_2 n^* \left[k^* f''(k^*) + f'\left(k^*\right)\right] + \gamma(n^*)V'\left(k^*\right) = 0 \tag{12}$$

and applying the envelop theorem to the maximization problem in (7) we also have:

$$V'\left(k^*\right) = \left[(1-\tau)w'\left(k^*\right) + (1-\delta)\right]u_1 \tag{13}$$

Thus we have equations (10)–(13) to solve for the steady-state solutions. We can substitute (10) in (11) and (13) in (12) to eliminate V and V' and then we will have two equations in two unknowns, k^* and n^*. Suppose u and f are twice continuously differentiable and all the conditions of the implicit function theorem are satisfied so that from (10)–(11) we get $n(k) = \eta_1(k)$ and from (12)–(13) we get $n(k) = \eta_2(k)$. When these two graphs are plotted on $n - k$ axes, the points at which these two curves intersect are the steady-state solutions, k^* and n^*.

In the rest of the section by assuming simpler functional forms for analytical derivations and more general functional forms for numerical derivations, we determine the signs of the following:

$$\frac{\partial k^*}{\partial \theta}, \frac{\partial k^*}{\partial \tau}, \frac{\partial n^*}{\partial \theta}, \frac{\partial k^*}{\partial \tau}.$$

Example 1: Let the utility function be of the type that the marginal utility of first period consumption is constant. Let us further assume that $\gamma(n)$ is constant and the production function is of the Cobb-Douglas form as follows:

$$U(c_t^t, c_{t+1}^t) = c_t^t + \beta \log c_{t+1}^t$$
$$\gamma(n) = \gamma_0, \quad f(k) = Ak^{\sigma}, \; 0 < \sigma < 1 \tag{14}$$

It is easy to derive the following values from (eq 9)–(eq 12):

$$n^* = \frac{\beta\sigma}{(\theta + k^*)(\sigma + \tau(1-\sigma))} \tag{15}$$

$$\gamma_0 \left(\sigma + \tau(1-\sigma)\right)\left[(1-\tau)\sigma(1-\sigma)Ak^{*\sigma} + (1-\delta)k^*\right] = \beta\sigma\left[\frac{k^*}{\theta + k^*} - \sigma\right] \tag{16}$$

The left hand side of (16) is a concave increasing function of k^* which is zero when $k^* = 0$ and goes to ∞ as $k \to \infty$, the right hand side is also an increasing concave function of k^* which takes value $-\beta\sigma^2$ when $k = 0$ and it goes to $\beta\sigma(1 - \sigma)$ as $k \to \infty$; further, the right hand side is independent of τ. The solution of this equation determines the optimal steady-state k^* values. From (15), the steady-state n^* is determined. Since the steady-state solutions for k^* are points where two increasing concave curves mentioned above intersect, there are at most two solutions. For instance, assuming the parameter values $\gamma_0 = 0.15$; $\sigma = .35$; $A = 1$; $\delta = .25$; $\theta = .35$; $\beta = .85$ and $\tau = 0$ or $\tau = .05$, there are two steady-state solutions for k^* namely, $k^* = .2392$, $n^* = 1.443$ and $k^* = 3.7886$, $n^* = .2053$ when $\tau = 0$; $k^* = .2439$, $n^* = 1.3095$ and $k^* = 3.4072$, $n^* = .2262$ when $\tau = .05$.

In general we cannot determine which way the left hand-side curve moves when τ increases from the value $\tau = 0$. Let us further assume that $\delta = 1$. In this case, it is clear that the curve moves to the right as τ increases from $\tau = 0$; the introduction of social security increases the steady-state capital labour ratio. This result is true even when δ is close to 1. For instance, suppose $\gamma_0 = 0.15$; $\sigma = 0.35$; $A = 1$; $\delta = 0.9$; $\theta = .35$; $\beta = .85$; when $\tau = 0$ then $k^* = 0.21185$ and $n^* = 1.5128$ and when $\tau = .05$, then $k^* = .213035$ and $n^* = 1.5097$. The above are the only steady-state solutions. For these economies with a sufficiently high value for δ, it follows that $\partial k^*/\partial\tau > 0$ and $\partial n^*/\partial\tau < 0$. In this simple economy, the effect of social security would be to reduce the fertility level and increase the capital labour ratio and hence per capita income in the long-run.

The comparative statics with respect to child rearing cost θ is simpler. Note that the left hand side of (16) does not depend on θ; as θ increases, the curve represented by the right hand side of (16) shifts to the right with the two end-points fixed. Thus we have that $\frac{\partial k^*}{\partial\theta} > 0$. From equation (15) it follows that $\frac{\partial n^*}{\partial\theta} < 0$.

The same comparative static results hold in models with non-linear utility and discount factor functions that are derived by perturbing the functions slightly in (14).

Example 2: We consider now a case where utility function, production function and discount factor functions are all strictly concave.

$$u(c_1, c_2) = \alpha \frac{c_1^{1-\rho}}{1-\rho} + \beta \frac{c_2^{1-\rho}}{1-\rho}, \quad \rho \neq 1,\ \rho > 0,\ 0 < \alpha, \beta < 1 \tag{17}$$

$$f(k) = k^{\sigma}, \quad 0 < \sigma < 1 \tag{18}$$

$$\gamma(n) = \gamma_0 n^{1-\gamma_1},\ 0 < \gamma_0 < 1,\ 0 \leq \gamma_1 \leq 1 \tag{19}$$

We further calibrate the model by assuming the following parameter values: $A = 100$, $\rho = 0.04$, $\sigma = 0.3$, $\gamma_0 = 0.15$, $\gamma_1 = 0.8$, $\theta = 31$, $\delta = 0.3$. The results are shown in Figures 1, 2 and 3.

We also carried out the simulation exercise for fixed social security taxes and varying child cost. The graphs are not shown here to save space. The main findings of this numerical exercise are the following:

1. While n^* is a decreasing function of τ, s^* first decreases and then it increases as τ increases. $V(k^*)$ decreases until the tax rate reaches roughly about 0.47 and then it sharply increases with the higher social security tax rates. However, population growth rate is negative in the latter region.

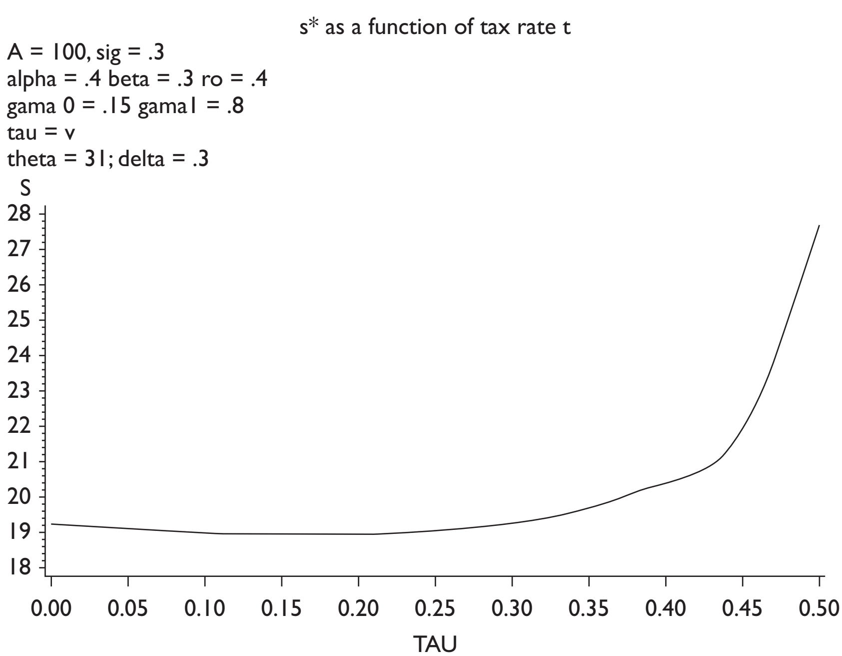

Figure 1. Steady-state Savings s^* as a Function of Social Security Tax Rate TAU

Source: Authors.

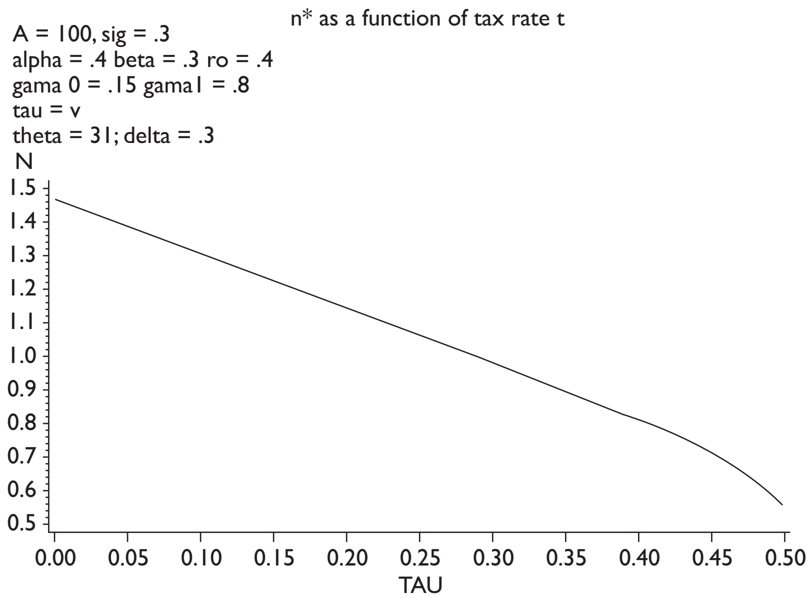

Figure 2. Steady-state Savings s^n as a Function of Social Security Tax Rate TAU

Source: Authors.

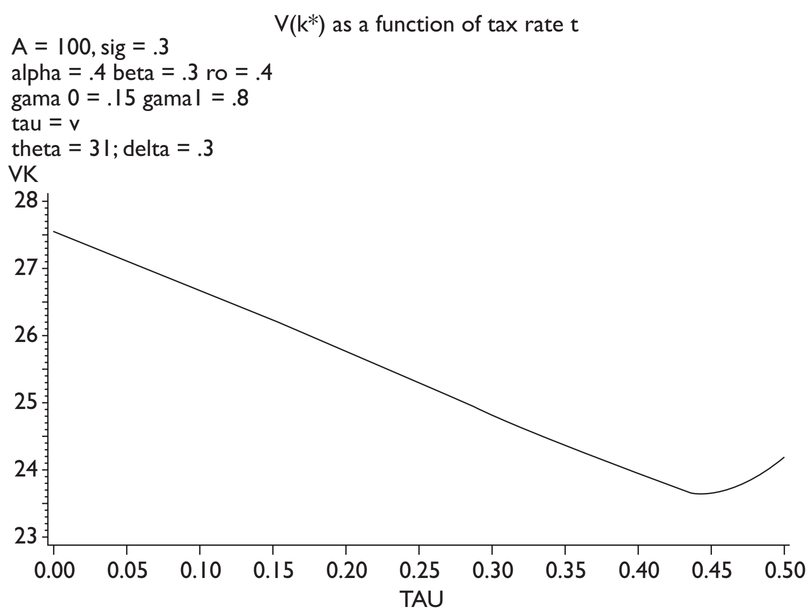

Figure 3. Steady-state Savings V^* as a Function of Social Security Tax Rate TAU.

Source: Authors.

2. n^* is a decreasing and s^* and $V(k^*)$ both are increasing functions of child cost θ.
3. The nature of these relationships is robust to most parameter values.

From both analytical derivation and the numerical exercise it appears that the long-run effect of introducing a social security programme is to have lower fertility and higher capital labour ratio. While it is not possible to derive analytically the effect of social security on saving and welfare even for the simplest functional forms that we have assumed, the numerical exercise, however, shows that both effects are negative and the effect on savings is rather very small unless the social security tax rate is very high. The negative effect of social security on fertility is found in other theoretical analyses of social security (see Raut, 1991, 1992; Nishimura and Zhang, 1992) and is also consistent with the general consensus of the empirical findings in developing countries.

The negative effect of social security on saving is derived in frameworks in which children are treated as poor man's capital (Raut, 1992: section 4). However, the effect of social security on savings is found to be positive in models where agents care about parent's old-age consumption and thus inducing parents to save little in the absence of social security so that parents could extract higher transfers from their children. The empirical findings on the effect of social security on savings are controversial, see Barro and Feldstein (1978) for a debate on this and very little is known empirically regarding the simultaneous effects on fertility and saving (see Nugent (1985) for a summary of empirical findings). The negative welfare effect of social security in our model is the opposite of what is found in latter models.

We should point out the well-known danger of steady-state comparisons to determine the long-run effect of social security τ and child cost θ, since it does not tell us the nature of the effects along the transition path to the new steady-state. For instance, the fact that steady-state welfare is lower in an economy with social security does not mean introducing social security is not welfare improving; the gains along the transitional path to the post social security steady-state could be enough to make social security worthwhile.[1]

Dynamics of the Optimal Path

It is not possible to study the dynamic properties of the optimal path for the general problem. Currently we only know a few results based on stronger assumptions. We summarize these results below. See Nishimura and Raut (1999) for proofs of these results and for further discussions. We assume that social security benefits are internalized by the agents in their decision making and that depreciation rate $\delta = 1$.[2] Define

$$\begin{aligned} W(k_t, k_{t+1}, n_t) \overset{def}{\equiv} \quad & u((1-\tau)w(k_t) - (\theta + k_{t+1})n_t, \\ & n_t R(k_{t+1}) + \gamma(n_t)V(k_{t+1})) \end{aligned} \tag{20}$$

and

$$n(k_t, k_{t+1}) \overset{def}{\equiv} \arg\max W(k_t, k_{t+1}, n_t) \tag{21}$$

Denote by

$$\bar{W}(k_t, k_{t+1}) \overset{def}{\equiv} W(k_t, k_{t+1}, n(k_t, k_{t+1})) \tag{22}$$

The original problem in (7) of finding $\{n_t, k_{t+1}\}_0^\infty$ is now equivalent to solving (21) and the following:

$$V(k_t) = \underset{\{k_{t+1}\}}{Max} \quad \bar{W}(k_t, k_{t+1}) \tag{23}$$

Suppose the above problem has a unique solution, denoted as $k_{t+1} = P(k_t)$. This function is known as policy function. The dynamics of the optimal path depends on the shape of $\bar{W}$ as stated in the following theorem. See Nishimura and Raut (1999) for a proof of this result and see Benhabib and Nishimura (1989) for an alternative proof.

Theorem 1: Let $\{k_t\}_0^\infty$ be an interior optimal solution of the problem (4) with $k_0 \neq k^*$, then the following are true:

$$\text{(i) } \bar{W}_{21} < 0 \Rightarrow (k_t - k_{t+1})(k_{t+1} - k_{t+2}) < 0$$

$$\text{(ii) } \bar{W}_{21} > 0 \Rightarrow (k_t - k_{t+1})(k_{t+1} - k_{t+2}) > 0$$

$$\text{(iii) } \bar{W}_{21} = 0 \Rightarrow k_{t+2} = k^* \text{ for all } t \geq 0.$$

It is not possible in general to determine the sign of the above cross partial derivatives of $\bar{W}$. We impose restrictions on the forms of the utility function $u(.,.)$, the altruism function $\gamma(n)$ along the lines of the available results in the literature to determine the sign of the $\bar{W}_{21}$ partial derivative and hence the dynamics of the equilibrium path.

Constant Marginal Utility of Young Age Consumption

Let the utility function be given by $u(c_t^t, c_{t+1}^t) = c_t^t + \psi(c_{t+1}^t)$, that is, the marginal utility of first period consumption is constant. We then have the following result:

Theorem 2: In economies with a Marshallian utility function, the optimal sequence of capital labour ratio, $\{k_t\}_0^\infty$ reaches the steady-state at $t = 1$.

It follows from the above theorem that the optimal fertility level, n_t also reaches steady-state at $t = 1$ and thus we note that this economy has a unique steady-state.

In the Barro and Becker (1989) one-period lived agent framework, the above result is true for Cobb-Douglas $\gamma(n)$. In our two-period lived agent framework the result is true for any general functional form for $\gamma(n)$, provided we restrict the utility function to be Marshallian.

Constant Discount Rate for Progeny's Welfare

In this section, we consider the case when $\gamma(n) = \gamma$, where $1 > \gamma > 0$, and characterize dynamic properties of optimal paths in terms of properties of felicity index function, $u(c_t^t, c_{t+1}^t)$. We extend the one period lived agent framework of Nishimura–Kunapongkul (1991), Kemp–Kondo (1986) and Lapan–Enders (1990) to two period lived agents framework. Since by assuming $\gamma(n_t) = \gamma$, agents in these models are assumed to care about welfare of a representative child, in one-period lived agent framework previous models incorporate motive for children by assuming the utility function u depends on c_t^t and n_t. In the two-period lived framework with life-cycle utility functions, an alternative motive for children has been studied in the literature by assuming that each child transfers a fraction, a, $0 < a < 1$, of their income to their parents (Bental, 1989; Neher, 1971; Raut, 1991, 1992; Willis, 1980). Since the first type of motive makes the dynamics technically intractable in our framework, we analyze the dynamics of equilibrium path along the second line of research. We, however, allow the agents to be altruistic towards their children. We further assume that $u_{12} > 0$ and $f'(k) + f''(k)k > 0$.

To state the most important assumption, let I denote the steady-state level of earnings of an adult and μ be the amount of resources received at old-age from each child. Consider the following life-cycle utility maximization problem by a representative agent who takes I, $(\theta + k)$, and μ as given to solve:

$$\max_{\{n\}} u(c_1, c_2)$$

$$\textit{subject to}$$

$$c_1 + (\theta + k)n = I$$

$$c_2 = n\mu$$

Denote the income elasticity of the demand for children, $e_n \equiv \frac{I}{n}\frac{dn}{dI}$ and the income elasticity of the marginal utility of income, $e_\lambda \equiv -\frac{I}{\lambda}\frac{d\lambda}{dI}$ for the above utility maximization problem. We have the following theorem:

Theorem 3: Let $k_0 \neq k^*$, then we have:

(i) $e_\lambda > e_n \Rightarrow \{k_t\}$ is monotonic.
(ii) $e_\lambda = e_n \Rightarrow k_t = k^*$ for all $t \geq 2$ and $n_t = n^*$ for all $t \geq 1$.
(iii) $e_\lambda < e_n \Rightarrow \{k_t\}$ and $\{n_t\}$ are oscillatory.

We do not know how $\{n_t\}$ will behave when $e_\lambda > e_n$.

Dynasty of One Period Lived Agents

Most growth models of endogenous fertility and savings in the dynastic framework assume one period lived agents (Barro and Becker, 1989; Becker and Barro, 1988; Benhabib and Nishimura, 1989 and others). Specializing our unified framework to this case, we have the dynamic properties of the optimal path characterized in terms of the economic parameter $e(n)$,

$$e(n) \equiv \frac{-n}{\left(\frac{\gamma'(n)}{\gamma(n)}\right)} \frac{d\left(\frac{\gamma'(n)}{\gamma(n)}\right)}{dn} \tag{24}$$

as stated in the following theorem:

Theorem 4: (i) If $e(n) < 1$, $\{k_t\}_0^\infty$ is monotone.
(ii) If $e(n) = 1$, $\{k_t\}_0^\infty$ reaches steady-state at $t = 1$.
(iii) If $e(n) > 1$, $\{k_t\}_0^\infty$ oscillates.

Let $k_0 \neq k^*$. If $e(n) = 1$, $\{n_t\}$ reaches steady-state at $t = 1$. If $e(n) < 1$, $\{n_t\}$ is oscillatory. Barro-Becker (1989) assumed Cobb-Douglas form for $\gamma(n)$ which forces $e(n) = 1$

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Notes

1. We are grateful to T.N. Srinivasan for pointing out this to us.
2. Bental (1989), Cignio and Rosati (1992) also made similar assumptions regarding the effect of social security on individual choices. Alternatively, we can view such type of social security transfers as within family intergenerational transfers where the fraction of income that agents transfer to their old parents is exogenously determined (for example, in Neher (1971), Raut (1991,1992) and Willis (1980)).

References

Barro, Robert J. & Feldstein, M. (1978). *The impact of social security on private saving: Evidence from the U.S. time series*. Washington, DC: American Enterprise Institute.

Barro, Robert J., & Becker, Gary S. (1989). Fertility choice in a model of economic growth. *Econometrica*, *57*(2), 481–501.

Becker, G.S., & Barro, R.J. (1988). A reformulation of the economic theory of fertility. *Quarterly Journal of Economics*, *CIII*(1), 1–25.

Benhabib, Jess & Nishimura, Kazuo (1989). Endogenous fluctuations in the Barro-Becker theory of fertility. In A. Wenig & K.F. Zimmermann (Eds), *Demographic change* *and economic development, studies in contemporary economics* (pp. 29–41). Berlin, Heidelberg: Springer.

Bental, Ben (1989). The old-age security hypothesis and optimal population growth. *Journal of Population Economics*, *1*, 285–301.

Cignio, A., & Rosati, F. (1992). The effects of financial markets and social security on savings and fertility behaviour in Italy. *Journal of Population Economics*, *5*(4), 319–341.

Easterlin, Richard (1987). *Birth and Fortune*, Second edition. Chicago: University of Chicago Press.

Eckstein, E., & Wolpin, K. (1985). Endogenous fertility and optimal population size. *Journal of Public Economics*, *27*(1), 93–106.

Kemp, M.C., & Kondo, Hitoshi (1986). Overlapping generations, competitive efficiency and optimal population. *Journal of Public Economics*, *30*, 237–247.

Lapan, Harvey E., & Enders, W. (1990). Endogenous fertility, Ricardian equivalence, and debt management policy. *Journal of Public Economics*, *41*, 227–248.

Neher, P.A. (1971). Peasants, procreation and pensions. *American Economic Review*, *61*, 380–389.

Nishimura, K. & Kunaponagkul V. (1991). Economic growth and fertility rate cycles. *Chaos, Solutions and Fractals*, *1*, 475–484.

Nishimura, Kazuo & Raut, L.K. (1999). Endogenous fertility and growth dynamics. In Gustav Ranis & Lakshmi K. Raut (Eds), *Trade, growth and development: Essays in honor of professor T.N. Srinivasan* (pp. 39–53). New York: Elsevier.

Nishimura, Kuzuo & Zhang, Junsen (1992). Pay as you go public pensions with endogenous fertility. *Journal of Public Economics*, *48*, 239–258.

Nugent J. (1985). The old-age security motive for fertility. *Population and Development Review*, *11*(1), 75–97.

Raut, L.K. (1991). Capital accumulation, income distribution and endogenous fertility in an overlapping generations general equilibrium model. *J. Development Economics*, *34*(1/2), 123–150.

———. (1992). Effects of social security on fertility and saving: An overlapping generations model. *Indian Economic Review*, *27*(1), 25–43.

———. (1992b). Subgame perfect manipulation of children by overlapping generations of agents with two-sided altruism and endogenous fertility. Discussion Paper No. 92–54, University of California, San Diego.

Raut, L.K., & Srinivasan, T.N. (1994). Dynamics of endogenous growth. *Economic Theory*, *4*, 777–790.

Veall, Michael R. (1986). Public pensions as optimal social contracts. *Journal of Public Economics*, *31*, 237–251.

Willis, R. (1980). The old-age security hypothesis and population growth. In Burch (Ed.), *Demographic behavior: Interdisciplinary perspectives on decision making*. Boulder: Westview Press.