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Theories of long-run growth: old and new

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1 Introduction

There has been a recent revival in theorizing about long-run growth after a hiatus of over two decades since the last spurt in the 1950s and 1960s. The latter was itself inspired much earlier by the pioneering works of Frank Ramsey (1928) on optimal saving and of von Neumann (1945) on balanced growth at a maximal rate, and also to dynamic extensions of the Keynesian model by Harrod (1939) and later by Domar (1947). In the largely neoclassical growth theoretic literature of the 1960s and earlier, one could distinguish three strands.

The first strand is *positive*, or, better still, *descriptive* theory aimed at explaining the stylized facts of long-run growth in industrialized countries (particularly the United States) such as a steady secular growth of aggregate output and relative constancy of the share of savings, investment, labor, and capital income in aggregate output. These stylized facts themselves had been established by the work of empirically oriented economists such as Abramovitz (1956), Denison (1962), and Kuznets (1966), who were mainly interested in accounting for observed growth. Solow's (1956, 1957) celebrated articles and later work by Jorgenson and Griliches (1966) and others are examples of descriptive growth theory and related empirical analysis. Uzawa (1961, 1963) extended Solow's descriptive one-sector model into a two-sector model. As Stiglitz (1990) remarked, by showing that the long-run steady state growth rate could be unaffected by the rate of savings (and investment) and that, even in the short run, the rate of growth was mostly accounted for by the rate of labor-augmenting technical progress, Solow challenged then conventional wisdom.

The second strand is *normative* theory which drew its inspiration from Ramsey's (1928) classic paper on optimal saving. In contrast with the descriptive models in which the aggregate savings rate was exogenously specified (usually as a constant over time), the normative models derived time-varying savings rates from the optimization of an intertemporal social welfare function. There were mainly two variants of such normative models: one-sector models (e.g. Cass, 1965; Koopmans, 1965) and two-sector models (Srinivasan, 1962, 1964; Uzawa, 1964). The contribution of Phelps (1961) is also normative, but it focused only on the steady state level of consumption per worker rather than on the entire transitional time path to the steady state and solved for that savings rate which maximized the steady state level of consumption per worker.

The third strand of theory is primarily neither descriptive nor normative although it is related to both. Harrod's dynamic extension of the Keynesian model (with its constant marginal propensity to save) raised the issue of stability of the growth path by contrasting two growth rates: the *warranted* rate of growth that would be consistent with maintaining the savings–investment equilibrium and the *natural* growth rate as determined by the growth of the labor force and technical change. In this model, unless the economy's behavioral and technical parameters keep it on the knife edge of equality between warranted and natural growth rates, there would be either growing under-utilization of capacity if the warranted rate exceeds the natural rate or growing unemployment if the natural rate exceeds the warranted rate. Indeed this knife-edge property resulting from Harrod's assumption that capital and labor are used in fixed proportions led Solow to look for growth paths converging to a steady state by replacing Harrod's technology with a neoclassical technology of positive elasticity of substitution between labor and capital.

von Neumann's (1945) model is also part of the third strand. In this model production technology is characterized by a finite set of constant returns to scale activities with inputs being committed at the beginning and outputs emerging at the end of each discrete production period. There are no non-produced factors of production such as labor or exhaustible natural resources. In the "primal" version, von Neumann characterized the vector of activity levels that permitted the maximal rate of *balanced growth* (i.e. growth in which outputs of all commodities grew at the same rate) given that the outputs of each period were to be ploughed back as inputs in the next period. In the "dual" version, a vector of commodity prices and an interest rate were derived which had the properties that the value of the output of each activity was no higher than the value of inputs inclusive of interest and that the interest rate was the lowest possible. Under certain assumptions about the technology von Neumann showed that, first, the maximal growth rate of output of the primal was equal to the (minimal) interest rate associated with the dual, and second, the usual complementary slackness relations obtained between the vector of activity levels, prices, growth, and interest rates.

Although *prima facie* there is no normative rationale for balanced growth and the maximization of the growth rate, particularly in a set-up with no final consumption of any good, it turned out that the von Neumann path of balanced growth at the maximal rate has a "normative" property. As Dorfman et al. (1958) conjectured and Radner (1961) later rigorously proved, given an objective that is a function only of the terminal stocks of commodities, the path starting from a given initial vector of stocks that maximizes this objective would be "close" to the von Neumann path for "most" of the time, as long as the terminal date is sufficiently distant from the initial date regardless of the initial stocks and of the form of the objective function. This so-called "turnpike" feature was later seen in other growth models in which final consumption is allowed and production involves the use of non-produced factors. For example, in the Koopmans–Cass model in which the objective is to maximize the discounted sum of the stream of utility of per capita consumption over time, a unique steady state exists which is defined by the discount rate, the rate of growth of the labor force and the technology of production. *All* optimal paths, i.e. paths that maximize the objective function and start from different initial conditions, converge to this steady state regardless of the functional form of the utility function. As such all optimal paths stay "close" to the steady state path for "most" of the time.

Barring a few exceptions to be noted below, in the neoclassical growth models production technology was assumed to exhibit constant returns to scale and in many, though not all, models smooth substitution among inputs with strictly diminishing marginal rates of substitution between any two inputs along an isoquant was also posited. Analytical attention was focused on conditions ensuring the existence and uniqueness of steady state growth paths along which all inputs and outputs grew at the same rate – the steady state being the path to which all transitional paths starting from any given initial conditions and satisfying the requirements of specified descriptive rates of accumulation or of intertemporal welfare optimality converged. The steady state growth rate was the *exogenous* rate of growth of the labor force in efficiency units so that, in the absence of (exogenous) labor-augmenting technical progress, output per worker was constant along the steady state.

Turning to the exceptions, Solow (1956) himself drew attention to the possibility that a steady state need not even exist and, if one existed, it need not be unique. Indeed output per worker could grow indefinitely even in the absence of labor-augmenting technical progress if the marginal product of capital was bounded below by a sufficiently high positive number. Helpman (1992) also draws attention to this. In addition, there could be multiple steady states some of which would be unstable if the production technology exhibited nonconvexities. We return to these issues below.

There were also exceptions to the exogeneity of technical progress and the rate of growth of output along a steady state. In the one-sector, one-factor models of Harrod and Domar and the two-sector models of Feldman (1928, as described in Domar, 1957) and Mahalanobis (1955) marginal capital–output ratios were assumed to be constant so that by definition the marginal product of capital did not decline. Growth rate was *endogenous* and depended on the rate of savings (investment) in such one-sector models and on the allocation of investment between sectors producing capital and sectors producing consumer goods in the two-sector models. Kaldor and Mirrlees (1962) endogenized technical progress (and hence the rate of growth of output) by relating productivity of workers operating newly produced equipment to the rate of growth of investment per worker. And there was the celebrated model of Arrow (1962) of "learning by doing" in which factor productivity was an increasing function of cumulated output or investment. Uzawa (1965) also endogenized technical progress by postulating that the rate of growth of labor-augmenting technical progress was a concave function of the ratio of labor employed in the education sector to total employment. The education sector was assumed to use labor as the only input. Uzawa's model has influenced recent contributions to growth theory.

The recent revival of growth theory started with the influential papers of Lucas (1988) and Romer (1986). Lucas motivated his approach by arguing that neoclassical growth theory cannot account for observed differences in growth across countries and over time and its evidently counter-factual prediction that international trade should induce rapid movements toward equality in capital–labor ratios and factor prices. He argued that

> in the absence of differences in pure technology then, and under the assumption of no factor mobility, the neoclassical model predicts a strong tendency to income equality and equality in growth rates, tendencies we can observe within countries and, perhaps, within the wealthiest countries taken as a group, but which simply cannot be seen in the world at large. When factor mobility is permitted, this prediction is powerfully reinforced.
>
> (Lucas, 1988, pp. 15–16)

He then goes on to suggest that the one factor isolated by the neoclassical model, namely variation across countries in technology,

> has the potential to account for wide differences in income levels and growth rates . . . when we talk about differences in "technology" across countries we are not talking about knowledge in general, but about the knowledge of particular people, or particular subcultures of people. If so, then while it is not exactly wrong to describe these differences (as) exogenous . . . neither is it useful to do so. We want a formalism that leads us to think about individual decisions to acquire knowledge, and about the consequences of these decisions for productivity.

He draws on the theory of "human capital" to provide such a formalism: each individual acquires productivity-enhancing skills by devoting time to such acquisition and away from paying work. The acquisition of skills by a worker not only increases her productivity but, by increasing the average level of skills in the economy as a whole, it has a spill-over effect on the productivity of all workers by increasing the average level of skills in the economy as a whole.

Romer also looked for an alternative to the neoclassical model of long-run growth to escape from its implications that "initial conditions or current disturbances have no long-run effect on the level of output and consumption . . . in the absence of technical change, per capita output should converge to a steady-state value with no per capita growth" (Romer, 1986, pp. 1002–3). His is "an equilibrium model of endogenous technological change in which long-run growth is driven primarily by the accumulation of knowledge by forward-looking, profit-maximizing agents" (p. 1003). While the production of new knowledge is through a technology that exhibits diminishing returns, "the creation of new knowledge by one firm is assumed to have a positive external effect on the production possibilities of other firms . . . [so that] production of consumption goods as a function of stock of knowledge exhibits increasing returns; more precisely, knowledge may have an increasing marginal product" (p. 1003).

It should be noted that the spill-over effects of the average stock of human capital per worker in the Lucas model and of knowledge in the Romer model are externalities unperceived (and hence not internalized) by individual agents. However, for the economy *as a whole* they generate increasing scale economies even though the perceived production function of each agent exhibits constant returns to scale. Thus by introducing nonconvexities through the device of a Marshallian externality Lucas and Romer were able to work with intertemporal competitive (albeit a socially nonoptimal) equilibrium. Both in effect make assumptions that ensure that the marginal product of physical capital is bounded away from zero and as such it is not surprising that in both models sustained growth in income per worker is possible. Thus both avoid facing the problem[1] that research and development (R&D) that lead to technical progress are "naturally associated with imperfectly competitive markets, as Schumpeter (1942) had forcefully argued" (Stiglitz, 1990, p. 25). Later work by others (e.g. Grossman and Helpman, 1991) formulated models in which firms operating in imperfectly competitive markets undertook R&D.

The literature on growth theory has grown by leaps and bounds in the 1980s. It is not our purpose to survey this literature critically. Instead we consider a few *selected* models that address the issues of long-run sustained growth in per capita income, possible multiplicities in long-run equilibria with different growth rates and convergence or otherwise to steady states where they exist. The models are couched in three alternative frameworks within the neoclassical paradigm: descriptive growth *à la* Solow (1956), optimal growth with infinitely lived agents *à la* Ramsey–Cass–Koopmans and finally the finitely lived overlapping generations *à la* Samuelson (1958) and Diamond (1965). Section 2 briefly reviews neoclassical growth models to set the stage for a discussion in section 3 of models that generate sustained long-run growth with possible multiple growth equilibria. Section 4 takes another approach to endogenous growth by assuming that population density has an external effect on the production process so that fertility decisions of individual households determine the dynamic evolution of production possibilities endogenously. Unlike the recent growth literature, the model of section 4 is not geared to generating steady states and, in fact, its nonlinear dynamics generates a plethora of outcomes. Section 5 concludes the chapter.

2 Neoclassical growth models

2.1 Solow

The main motivation behind Solow's growth model, as mentioned earlier, was to explain the stability of the growth rates of US output during the first half of the twentieth century by means of a simple model. Solow assumes an aggregate production function

$$Y_t = A_t F(K_t, b_t L_t) \tag{1.1}$$

where Y_t is aggregate output at time t, K_t is the stock of capital, L_t is labor hours at time t, A_t ($A_0 \equiv 1$) is the disembodied technology factor (i.e. index of total factor productivity) so that output at time t associated with any combination of capital stock and labor input is A_t multiplied by that at time zero with the same combination. Analogously b_t (with $b_0 \equiv 1$) is the efficiency level of a unit of labor in period t so that a unit of labor at time t is equivalent to b_t units of labor at time zero. Thus the technical progress induced by increases in b_t is *labor augmenting*. It is easily seen that technical progress through A_t is Hicks neutral and that through b_t is Harrod neutral.

Let us denote by $\tilde{k}_t \equiv K_t/b_t L_t$ the ratio of capital to labor in efficiency units in period t, by $k_t = K_t/L_t$ the ratio of capital to labor in natural units, and by $y_t \equiv Y_t/b_tL_t$ the level of output or income per unit of labor in efficiency units. Solow made the following crucial assumptions.

Assumption 1 (Neoclassical)

F is homogeneous of degree one in its arguments and concave.

Given assumption 1, the average product of an efficiency unit of labor, i.e. $(1/b_tL_t)F(k_t, b_tL_t)$ equals $F(\tilde{k}_t, 1)$.

Let $f(\tilde{k}_t) = F(\tilde{k}_t, 1)$. Clearly concavity of F implies concavity of f as a function of $\tilde{k}_t$. In fact f is assumed to be strictly concave with $f(0) = 0$.

Assumption 2 (Inada)

$$\lim_{\tilde{k}\to 0} f'(\tilde{k}) = \infty \qquad \text{and} \qquad \lim_{\tilde{k}\to\infty} f'(\tilde{k}) = 0$$

In a closed economy, assuming that labor is growing exogenously as $L_t = (1 + n)^t L_0$, human capital or skill level is growing exogenously as $b_t = (1 + b)^t$, and capital depreciates at the rate δ per period, and denoting by c_t the level of consumption per efficiency unit of labor we have

$$\tilde{k}_{t+1} = \frac{A_t f(\tilde{k}_t) + (1 - \delta)\tilde{k}_t - c_t}{(1 + n)(1 + b)} \tag{1.2}$$

Solow further assumed that the savings rate is constant, i.e. $c_t = (1 - s)y_t$. Then (1.2) becomes

$$\tilde{k}_{t+1} = \frac{sA_t f(\tilde{k}_t) + (1 - \delta)\tilde{k}_t}{(1 + n)(1 + b)} \tag{1.3}$$

Equation (1.3) is the fundamental difference equation of the Solow model. If there is no disembodied technical progress so that $A_t = 1$ for all t, then the phase diagram of the dynamic system can be represented as in figure 1.1. It is clear from the figure that starting from any arbitrary initial capital–labor ratio $\tilde{k}_0 > 0$, as $t \to \infty$ the economy will converge to the steady state $\tilde{k}^* > 0$ in which all the per capita variables, including per capita income, will grow at the rate b.

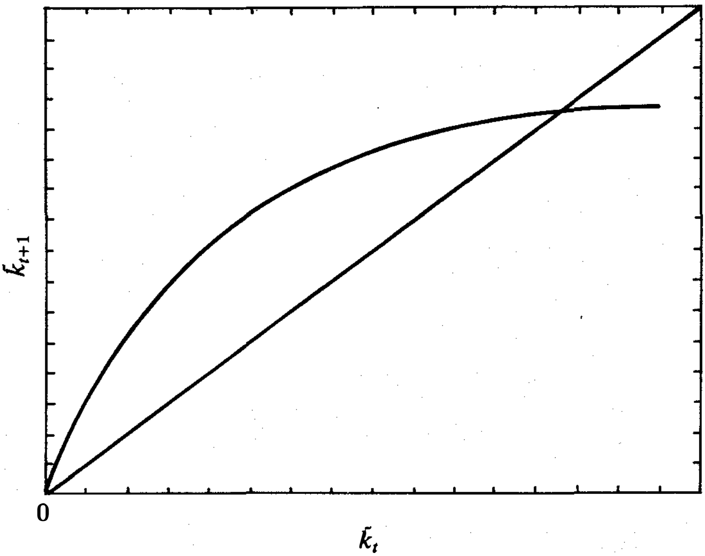


Figure 1.1 Phase diagram of Solow model.

Thus if $b = 0$ per capita income, consumption and savings do not grow along the steady state. Further, policies that permanently affect savings rate or fertility rate will have no long-run growth effects.

It is clear from figure 1.1, however, that out of the steady state (i.e. in the short run) economies will exhibit growth in per capita income even without technological change. The rate of growth will depend on the initial capital–labor ratio and the time period over which the average growth rate is calculated. It can be shown that the average growth rate *decreases* as the initial capital–labor ratio $\tilde{k}_0$ (and hence initial income per head) *increases*. As the initial capital–labor ratio tends to $\tilde{k}^*$, the average growth rate of per capita income converges to b, the exogenously given rate of labor-augmenting technical progress. This is indeed one of the convergence hypotheses that are tested in the recent empirical literature on growth. Policies that affect s and n clearly affect growth rates out of steady state. However, these growth effects are only temporary and the marginal product of capital will be declining over time. This predicted fall in the marginal product of capital is not observed in US historical data, however.

It was mentioned earlier that a primary goal of the recently revived growth theory is to build models that can generate sustained long-run growth in per capita income. A related objective is to ensure that the long-run growth rate of income (and in fact the entire time path of income) not only depends on the parameters of the production and utility functions but also on fiscal policies, foreign trade policies, and population policies. In most models of "new" theory, the primary goal is accomplished through increasing scale economies in the aggregate production. The resulting nonconvexities lead to multiple equilibria and hysteresis in some models so that history (i.e. initial conditions as well as any past shocks experienced by the economy) and policies have long-term effects.

It will be recalled from earlier discussion, however, that per capita output can grow indefinitely even in traditional growth models if the marginal product of capital is bounded away from zero as the capital–labor ratio grows indefinitely. Thus the standard neoclassical assumption that the marginal product of capital is a strictly decreasing function of the capital–labor ratio is not inconsistent with indefinite growth of per capita output. It has to diminish to zero as the capital–labor ratio increases indefinitely to preclude such growth. This is easily seen from equation (1.3).

Consider the simplest version of the neoclassical growth model with $b_t = 1$ and $A_t = 1$ for all t so that $\tilde{k}_t = k_t$. Let $f(0) = 0$ and let the marginal product of capital, i.e. $f'(k)$, be bounded away from $(n + \delta)/s$ (i.e. $f'(k) > (n + \delta)/s$ for all k). Strict concavity of $f(k)$ together with $f(0) = 0$ implies $f(k) > kf'(k) > [k(n + \delta)]/s$ so that from (1.3) it follows that $k_{t+1} > k_t$. This in turn implies that output per worker $f(k_t)$ grows at a positive rate at all t. Moreover, given strict concavity of $f(k)$ it follows that $f'(k)$ is monotonically decreasing, and hence has a limiting value as $k \to \infty$, say γ_y, that is at least as large as $(n + \delta)/s$. As such it can be verified that the asymptotic growth rate of output and consumption will be at least as large as $[s\gamma_y - (n + \delta)]/(1 + n) \geqslant 0$. The savings rate s can be made endogenous using the Samuelson–Diamond overlapping generations framework or the Ramsey–Cass–Koopmans infinitely lived agent framework, thus leading to a theory of endogenous growth. Thus the neoclassical framework can endogenously generate long-run growth in per capita income. However, the assumption that the marginal product has a positive lower bound is not particularly attractive since it implies that labor is not essential for production.[2]

2.2 Ramsey–Koopmans–Cass framework

The optimal growth literature derives the savings rate endogenously by assuming that there is an infinitely lived representative agent who maximizes an additive time-separable intertemporal welfare

$$\sum_{t=0}^{\infty} \rho^t u(\tilde{c}_t)$$

with respect to $\{\tilde{c}_t\}_0^\infty$ subject to the restriction (1.2) with $A_t \equiv 1$, where $u(.)$ is a twice continuously differentiable, strictly concave, and monotonic function. It is indeed odd that per period utility $u(.)$ is a function of consumption per *efficiency unit* of labor rather than of consumption per worker. Only analytical convenience dictates this choice. Under assumption 2, it can be shown that the set of feasible $\{\tilde{c}_t\}_0^\infty$ is compact and the above sum is a well-defined continuous[3] function of $\{\tilde{c}_t\}_0^\infty$. Thus, the above problem has a solution. Let us denote the relationship in (1.2) with $\tilde{c}_t \equiv 0$ and $A_t \equiv 1$ by the difference equation $\tilde{k}_{t+1} = \psi(\tilde{k}_t)$. Assuming that f satisfies the Inada condition, one can show there exists a unique positive fixed point $\bar{k}$ for ψ. Using dynamic programming techniques, one can show that the optimal capital accumulation path from any initial $\tilde{k}_0 < \bar{k}$ is given by a nondecreasing policy function $\tilde{k}_{t+1} = \pi(\tilde{k}_t) \geqslant \tilde{k}_t$. It can also be shown that an optimal $\{\tilde{k}_t\}_0^\infty$ with $\tilde{k}_0 \leqslant \bar{k}$ is a monotonic sequence bounded above, and hence $\tilde{k}_t$ converges to a limit point, say $\tilde{k}^* > 0$ as $t \to \infty$; $\tilde{k}^*$ satisfies the following:[4]

$$f'(\tilde{k}^*) = \frac{(1+n)(1+b)}{\rho} - (1-\delta) \tag{1.4}$$

It is clear that the limit point is unique. Since it depends only on the production function and the parameters n, δ, ρ, and b, it is independent of the utility function $u(.)$. Thus, for large t we have $k_t \approx \tilde{k}^* b_t$, i.e. for large t, optimal k_t, c_t, and y_t will be growing at constant rates[5] (in this case, all rates are equal to the rate of growth of b_t). This is the well-known turnpike result which states that starting from any initial capital–labor ratio the optimal path converges to the modified golden balanced growth path.

It also follows that if there is no Harrod-neutral technological change, i.e. $b = 0$, there is no growth in the capital–labor ratio and hence no growth in per capita income, and if $b > 0$, per capita income will be growing at the rate b.

It can be shown once again that, even when $b = 0$, there could still be growth in per capita income if the marginal product of capital is bounded away from zero. Moreover, the long-run growth rate in this case will depend on the rate of pure time preference ρ of the representative agent, the smaller the value ρ the larger being the rate of long-run growth. In so far as countries differ in ρ, their long-run growth rates will differ. In particular, if poverty is associated with impatience in the sense of a high value of ρ, then poor countries will have low growth rates. However, explaining intercountry differences in long-run growth entirely through differences in a parameter that represents tastes is not satisfactory since tastes need not be immutable but could be acquired.

2.3 Samuelson–Diamond overlapping generations framework

Although the overlapping generations framework was not developed by Samuelson and Diamond to examine growth issues, it turns out to be another useful approach to endogenizing savings. In addition it has all the basic features of the other two neoclassical growth frameworks discussed in sections 2.1 and 2.2. We briefly describe the framework and set up the notation for later use.

Assume that each agent lives for two periods, the first as a young person and the second as an old person. A young person of period t supplies one unit of labor, earns wages w_t, consumes c_t^t and saves s_t, taking the interest rate r_{t+1} between period t and $t + 1$ as given. In the next period he retires and finances his old-age consumption c_{t+1}^t with the returns from his savings while young. Formally, he maximizes his lifetime welfare $U(c_t^t, c_{t+1}^t)$ with respect to s_t subject to

$$c_t^t + s_t = w_t$$
$$c_{t+1}^t = (1 + r_{t+1})s_t$$

Denote the solution of the above problem by $H(w_t, 1 + r_{t+1})$. Assume that all markets are perfectly competitive, and producers are profit maximizers. For simplicity of exposition, we assume further that capital depreciates fully in one period and that capital has to be purchased a period ahead of its use in production. Then it follows from producer behavior that

$$\frac{w_t}{b_t} = f(k_t) - k_t f'(k_t) \equiv \omega(k_t) \text{ say} \tag{1.5}$$

$$1 + r_{t+1} = f'(k_{t+1}) \equiv R(k_{t+1}) \tag{1.6}$$

Substituting (1.5) and (1.6) in $H(,)$ and noting that $k_{t+1} = [(1 + n)(1 + b)]^{-1}s_t$, one can write the fundamental difference equation of the Samuelson–Diamond model as

$$k_{t+1} = \frac{H[\omega(k_t), R(k_{t+1})]}{(1 + n)(1 + b)} \tag{1.7}$$

If we specialize the functional form of the utility function to be Cobb–Douglas so that $U = \alpha \log c_t^t + (1 - \alpha) \log c_{t+1}^t$, then (1.7) becomes very similar to (1.3). Even for more general utility functions, most properties of the Solow model remain valid in this framework as well.

3 Models generating sustained long-run growth and multiple equilibria

3.1 Increasing returns

At the outset a distinction should be made between generating *sustained growth* in output per head and *endogenizing* the rate of growth. For example, with the production function $Y = K^a L^b$ where $0 < a, b < 1$ and $a + b > 1$ and with the labor force growing *exogenously* at the rate n there exists a unique steady state regardless of the savings rate in which output grows at the exogenous rate $n(a + b - 1)/(1 - a) > 0$. Thus increasing scale economies together with marginal product of capital strictly diminishing to zero (i.e. $0 <$

$a < 1$) leads to *sustained* but *exogenous* growth. On the other hand, constant returns to scale with marginal product of capital bounded away from zero at a sufficiently high positive value leads to *endogenous* and *sustained* growth. Thus increasing scale economies by themselves need not generate endogenous growth.[6] While keeping this in mind, it is important to distinguish how different types of increasing returns to scale in aggregate production arise in various growth models. We consider only two types: locally increasing marginal product of capital and scale economies due to spill-over effects. For simplicity of exposition, we assume in this section that $L_t \equiv 1$, $A_t = 1$, $b_t = 1\ \forall t \geqslant 0$. The first type arises when the marginal product of capital $f'(k)$ first increases with k and then decreases, or more generally when $f''(k) = 0$ has more than one but a finite number of solutions.

The second type arises in the models of Lucas and Romer. Building upon the work of Arrow (1962) and Sheshinski (1967), Romer (1986) considers an economy in which there are n identical firms; each has a production function of the form $Y_i = G(K_i, L_i, K)$ where K_i is the stock of knowledge capital or R&D capital employed by firm i and $K = \Sigma_{i=1}^{n} K_i$, the industry level aggregate stock of knowledge, and L_i is labor or any other inputs. K is assumed to have a positive spill-over effect on the output of each firm although the choice of K is external to the firm. Romer assumes that, for fixed K, G is homogeneous of degree one in other inputs. Supposing that all identical firms choose identical inputs, we can write $Y_i = G(K_i, L_i, nK_i)$. Define $F(K_i, L_i) \equiv G(K_i, L_i, nK_i)$. It is obvious that F exhibits increasing returns to scale in the inputs K_i and L_i. Again, besides those scale economies one needs to assume that the asymptotic marginal product of aggregate capital is positive to generate endogenous growth. Empirical support for the spill-over effect of R&D capital is found in several empirical investigations (see Bernstein and Nadiri (1989) on Canadian industry data, Jaffe (1986) on US manufacturing firm level data, and Raut (1991a) on Indian manufacturing firm level data).[7]

Following Romer, let us further assume that $L_i \equiv 1$ and denote the average product of labor by $f(k) = F(k, 1)$. Both types of increasing returns make $f(k)$ nonconcave and thus violate the neoclassical assumptions. The existence of a solution to optimal growth problems and turnpike results that were found to hold in all the neoclassical frameworks need not hold anymore. Instead, increasing returns open up the possibility for the marginal product of capital to be bounded away from zero, thus generating sustained long-run growth in these models. Moreover, the first type of increasing returns leads to multiple steady states, allowing history or the initial conditions to determine to which steady state the economy will converge. We illustrate these points with a brief discussion of a few contributions in the recent literature.

Broadly speaking given an appropriate choice of an infinite-dimensional commodity space and a topology such that the set of feasible consumption paths is compact and the social ordering is continuous, the existence of an optimal path is assured. For compactness of a feasible set some kind of bounding of the technology is necessary. Majumdar and Mitra (1983) assume that $f'(\infty) < 1 < f'(0) < \infty$ and that there exists a k_1 such that $f''(k_1) = 0$, $f''(x) > 0$ for $0 \leqslant x < k_1$, and $f''(x) < 0$ for $k_1 < x$. These assumptions imply that the marginal product of capital increases up to $k = k_1$ and then decreases. Somewhat more general assumptions are made by Majumdar and Nermuth (1982); they assume that $f'(\infty) < 1$ and also the following.

Assumption 3 (*Nonclassical*)

$f''(k) = 0$ has finitely many roots, and there exists $k_{\max} > 0$ such that $f(k_{\max}) = k_{\max}$, $f'(k) < 1$ for $k \geqslant k_{\max}$.

They show that there exists an optimal solution, and the turnpike results depend on the magnitude of the rate of time preference. Define $\bar{k} > 0$ to be a local modified golden rule if it is a local maximum of $\rho f(k) - k$ and $f(\bar{k}) > \bar{k}$. Let a steady state be any solution of $\rho f'(k) = 1$. A set of local modified golden rules could clearly be a proper subset of the set of steady states. Assume that an inflection point of $f(.)$ does not occur at a steady state, and investment is irreversible. For such an economy, if the discount factor ρ is not too large or too small, then there exist neighborhoods around each golden rule such that, depending on the neighborhood in which the initial capital–labor ratio lies, the optimal solution converges monotonically to the corresponding local golden rule. However, if ρ is too small, then all optimal programs converge to extinction, i.e. to $k = 0$ and $f(0) = 0$. If ρ is close to unity, all optimal solutions converge to the golden rule path with the largest k. It should be pointed out that the existence of multiple steady states and the dependence on the initial conditions for convergence of an optimal solution to a particular steady state are the consequences of the assumption that the production function exhibits increasing returns of the first type. In these models, there is no sustained long-run growth in any of the equilibria.

Romer (1986) posed the optimal growth problem in continuous time as follows:

$$\max_{\{c_t\} \geq 0} \int_0^{\infty} \exp(-\rho t)\, u(c_t)\, \mathrm{d}t$$

subject to

$$\frac{\dot{k}_t}{k_t} = h\left[\frac{g(k_t, nk_t) - c_t}{k_t}\right] \tag{1.8}$$

where $h(.)k_t$ represents the production function of "knowledge" capital. The *rate of growth* of knowledge is a function of resources devoted to its accumulation, i.e. savings as a proportion of the existing stock of knowledge. h is assumed to be concave and *bounded above* by a constant α. The latter ensures that asymptotically there are constant returns to aggregate capital. The production function $g(k_t, nk_t)$ for output (with n being the number of firms) is assumed to be globally *convex* as a function of k so that there are increasing returns. However, for a firm which treats the total knowledge stock $K_t \equiv nk_t$ as a parameter on which it has no influence, its production function $g(k, K)$ is assumed to be concave in k. Thus economy-wide stock of knowledge is a Marshallian externality to each firm. The solution to the optimization problem that takes into account the effect of k_t on *both* arguments of g (so that the externality is internalized) is socially optimal. By contrast, one could exogenously specify the second argument K_t of $g(,)$ and solve the optimal path for its first argument k_t. Of course the solution for k_t will in general depend on the exogenously specified path for K_t. By choosing that solution for which nk_t is equal to k_t for all t, one obtains the competitive equilibrium or privately optimal path.

For the existence of optimal solutions, Romer uses the following bounding conditions.

Assumption 4 There exist positive numbers μ and θ such that $g(k, nk) < \mu + k^{\theta}$.

He then shows that, if $\alpha\theta < \rho$, then the above problem has a socially optimal solution, and under some additional assumptions there also exists a competitive equilibrium solution.

As is to be expected, the social optimal cannot be supported as a competitive equilibrium without government intervention. In the absence of appropriate intervention (such as subsidies for private acquisition of knowledge financed by lump-sum taxation of consumers) each firm would choose to acquire less than the socially optimal amount of knowledge. Under assumptions that bound the social and private marginal product of capital from below by the discount rate ρ, Romer shows that k_t and c_t grow without bound in socially and privately optimal solutions.

3.2 Endogenous Harrod-neutral technological change and human capital

One obtains long-run growth in per capita income in standard neoclassical growth models with labor-augmenting technological change. Per capita income is given by $y_t = F(k_t, b_t)$, where F is as in equation (1.1) with the further assumption that $A_t \equiv 1$ for all t. If b_t is growing exogenously at a constant rate b, as long as k_t grows at the same rate in the long run the marginal product of capital remains constant and bounded away from zero. Thus in the long run with k_t and b_t growing at the rate b, y_t will also be growing at the rate b.

The role of human capital accumulation in Uzawa (1965) and Lucas (1988) is to endogenize Harrod-neutral (i.e. labor-augmenting) technological change. Let us briefly describe this mechanism following Lucas (1988). Suppose a worker of period t is endowed with b_t of human capital or skill and one unit of labor. He has to allocate his labor endowment between accumulating skills and earning wage income. If he devotes the fraction ϕ_t of his time to the current production sector and $1 - \phi_t$ (where $0 \leqslant \phi_t \leqslant 1$) to the learning sector (such as schooling or some vocational training program), he can increase his human capital in the next period by

$$\dot{b}_t = b_t\delta(1 - \phi_t) \tag{1.9}$$

It should be noted that the marginal return to time devoted to skill accumulation is constant and does not diminish. As Lucas himself points out, this is crucial for generating sustained growth per capita consumption in the long run. Since the opportunity cost of time spent on skill acquisition is forgone income that could have been used for consumption or accumulation of physical capital, this crucial assumption should be viewed as the equivalent of assuming that the marginal product of physical capital is constant as in the Harrod–Domar model.

The budget constraint for the representative agent is given by

$$c_t + \dot{k}_t = F(k_t, \phi_t b_t) - (n + \delta)k_t \tag{1.10}$$

From (1.10) it is clear that for given c_t and k_t, the agent faces a trade-off. He can spent more time currently (i.e. choose a larger ϕ_t) in the production sector and thus have a larger *current consumption* or *future physical capital*, or have a lower ϕ_t and thus have *larger future human capital* (i.e. higher b_t) and hence a *larger future stream of output*. It is clear that he would divide his savings between human capital and physical capital in a balanced way so that the marginal product of capital does not fall to zero. Under the further assumption that the production function is of the Cobb–Douglas form

$$F(K, L) = A(b_t)K_t^{\alpha}(b_t L_t)^{\beta} \qquad \alpha + \beta = 1 \qquad \alpha, \beta > 0$$

where the spill-over effect is given by $A(b_t) = Ab_t^{\mu}$, $0 < \mu$, it can be shown that along the balanced growth path the capital–labor ratio and hence per capita income and consumption will be growing at the rate

$$\gamma_y = \frac{1 - \beta + \mu}{1 - \beta}(1 - \phi)\delta$$

where ϕ_t is a constant equal to ϕ. Since γ_y is a function of ϕ which is endogenously determined, the growth rate of per capita income is endogenously determined. It should be noted that even if there is no spill-over effect, i.e. $\mu = 0$, γ_y is positive, and this of course is the consequence of the crucial assumption discussed above about the process of skill accumulation.

The Lucas model is essentially a two-sector growth model. Human capital and the process of its accumulation play essentially the same role as the capital goods sector in the two-sector model of Mahalanobis (1955). In this model marginal product of capital in the capital goods sector is constant – an assumption that is the equivalent of Lucas's crucial assumption about the process of human capital accumulation (Srinivasan, 1992). The rate of growth of income and consumption was endogenously determined in the Mahalanobis model by the share of investment devoted to the accumulation of capacity to produce capital goods. The share $1 - \phi_t$ of time devoted to skill acquisition plays an analogous role in the Lucas model.

Linearity of the technology of skill acquisition in the Lucas model is restrictive. It leads to a unique balanced growth solution. However, if a nonlinear (convex) technology is assumed, there could be multiple optimal balanced growth paths that are locally stable, as has been shown by Azariadis and Drazen (1990) in a Samuelson–Diamond overlapping generations model with endogenous human capital formation.

4 Agglomeration and congestion effects of population density and long-run growth

In Raut and Srinivasan (1991) we present a model that not only endogenizes growth and the process of shifts in production possibilities over time (i.e. technical change) but also generates richer dynamics than the models of recent growth theory. First, by assuming fertility to be endogenous,[8] we preclude the possibility of aggregate growth being driven solely by exogenous labor force growth in the absence of technical change. Second, by assuming that population density has an external effect (not perceived by individual agents) on the production process through either a negative conges tion effect or a positive effect in stimulating innovation and technical change, we make the change in production possibilities endogenously determined by fertility decisions of individual agents. However, unlike the new growth literature, our model, which is an extension of Raut (1985, 1991b), is not necessarily geared to generating steady states. In fact, the nonlinear dynamics of the model generates a plethora of outcomes (depending on the functional forms, parameters, and initial conditions) that include not only the neoclassical steady state with exponential growth of population with constant per capita income and consumption, but also growth paths which do not converge to a steady state and are even chaotic. Per capita output grows exponentially (and super exponentially) in some of the examples.

Our model draws on the insights of Boserup (1989) and Simon (1981) who, among others, have argued that the growth of population could itself induce technical change. In the Boserup model increasing population pressure on a fixed or very slowly growing supply of arable land induces changes in methods of cultivation, not simply through substitution of labor for land by choice of techniques within a known set of techniques but, more importantly, through the invention of new techniques. Simon also attributes a positive role for increases in population density in inducing technical progress. Since having a large population is not sufficient to generate growth (Romer, 1990), it is important to examine the mechanism by which population density influences innovation. However, neither of these two authors provides a complete theory of induced innovation. We do not provide one either: we believe that the inducement to innovate will depend largely on the returns and risks to resources devoted to innovative activity, and there is no particular reason to suggest that pre-existing relative factor prices or endowments will necessarily tilt these returns towards the search for technologies that save particular factors. Instead, we simply analyze the implications of assuming that technical change is influenced by population density (strictly speaking, population size) in a world where fertility is endogenous.

More precisely, we assume that technical change in our model economy is Hicks neutral and that its rate is determined by the change in the size of the working population. Thus, instead of the aggregate production function in equation (1.1), we use the following:

$$Y_t = A(L_t)F(K_t, L_t) \tag{1.11}$$

However, for both consumers and firms in this economy $A(L_t)$ is an externality. We introduce this externality in a model of overlapping generations in which a member of each generation lives for three periods, the first of which is spent as a child in the parent's household. The second period is spent as a young person working, having and raising children, and accumulating capital. The third and last period of life is spent as an old person in retirement living off support received from one's offspring and from the sale of accumulated capital. All members of each generation are identical in their preferences defined over their consumption in their working and retired periods. Thus, in this model the only reason that an individual would want to have a child is the support the child will provide during the parent's retired life. Production (of a single commodity which can be consumed or accumulated) is organized in firms which buy capital from the retired and hire the young as workers. Markets for product, labor, and capital are assumed to be competitive.

Formally, a typical individual of the generation which is young in period t has n_t children (reproduction is by parthenogenesis!), consumes c_t^t, c_{t+1}^t in periods t and $t + 1$, and saves s_t in period t. She supplies one unit of labor for wage employment. Her income from wage labor while young in period t is w_t and that is the only income in that period. A proportion α of this wage income is given to parents as old age support. While old in period $t + 1$, she sells her accumulated saving to firms and receives from each of her offspring the proportion α of his/her wage income. She enjoys a utility $U(c_t^t, c_{t+1}^t)$ from consumption. Thus her choice problem can be stated as

$$\max_{s_t, n_t > 0} U(c_t^t, c_{t+1}^t)$$

subject to

$$c_t^t + \theta_t n_t + s_t = (1 - \alpha) w_t \tag{1.12}$$

$$c_{t+1}^t = (1 + r_{t+1}) s_t + \alpha w_{t+1} n_t \tag{1.13}$$

where θ_t is the output cost of rearing a child while young.

Profit maximization of the producer yields (using the notation of section 2.3)

$$w_{t+1} = A(L_{t+1})[f(k_{t+1}) - k_{t+1} f'(k_{t+1})] \tag{1.14}$$

$$1 + r_{t+1} = A(L_{t+1}) f'(k_{t+1}) \tag{1.15}$$

In equilibrium, the private rates of return from investing in children and physical capital are equal so that arbitrage opportunities are ruled out. This implies that

$$\frac{\alpha w_{t+1}}{\theta_t} = 1 + r_{t+1} \tag{1.16}$$

Putting equations (1.14) and (1.15) in equation (1.16), we get an implicit equation linking k_{t+1}, θ_t, and α. It can be shown that under standard neoclassical assumptions on the production function, we can solve for k_{t+1} as a function $\Psi(\theta_t/\alpha)$. Since $k_{t+1} = s_t/n_t$ (given the assumption that capital depreciates fully in one generation), the budget constraints (1.12) and (1.13) become respectively $c_t^t = (1 - \alpha)w_t - S_t$ and $c_{t+1}^t = (1 + r_{t+1})S_t$, where $S_t = [\theta_t + \Psi(\theta_t/\alpha)]n_t$. S_t could be thought of as total savings.

Let us denote the solution of the above utility maximization problem as before by $S_t = H(w_t, 1 + r_{t+1})$. We can now express the solutions for n_t and s_t as

$$n_t = \frac{H(w_t, 1 + r_{t+1})}{\theta_t + \Psi(\theta_t/\alpha)}$$

and (1.17)

$$s_t = \frac{\Psi(\theta_t/\alpha)H(w_t, 1 + r_{t+1})}{\theta_t + \Psi(\theta_t/\alpha)}$$

Equation (1.17) determines the dynamics of the system. Let us first consider the simplest case in which child rearing cost $\theta_t = \theta$ for all $t \geqslant 0$. It is clear that $k_{t+1} = k^*$ for all $t \geqslant 1$ in this case. Assuming further that the utility function is Cobb–Douglas, i.e. $U = a \log c_t^t + (1 - a) \log c_{t+1}^t$, we have $H(w_t, 1 + r_{t+1}) = (1 - a)(1 - \alpha)w_t$. Equation (1.17) now yields

$$n_t = \frac{L_{t+1}}{L_t} = \frac{(1 - \alpha)(1 - a)}{\theta + k^*} w^* A(L_t)$$

or

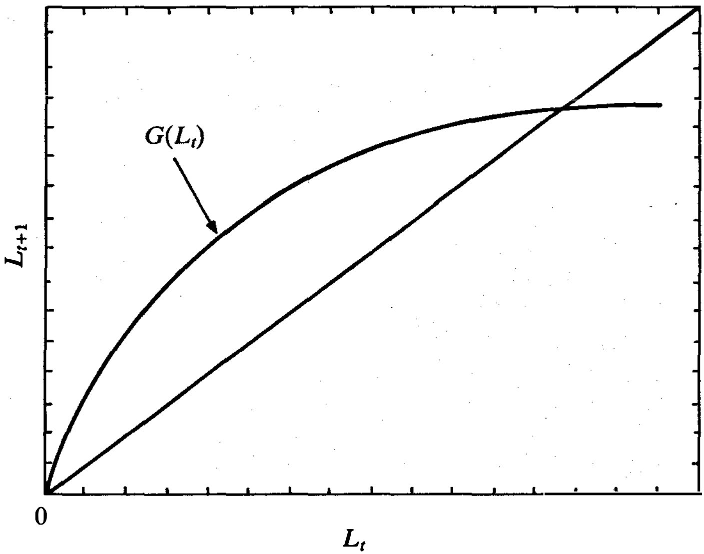
$$L_{t+1} = \lambda L_t A(L_t) \equiv G(L_t) \text{ say} \tag{1.18}$$

where $\lambda = [(1 - \alpha)(1 - a)w^*]/(\theta + k^*)$. From (1.11) we note that per capita income is given by $y_t = A(L_t)f(k^*)$. Thus, the dynamics of population long-run behavior of per capita income hinge on the form of $A(L_t)$. It should be recalled that although the fertility decisions of individuals determine L_t and hence $A(L_t)$, this is an unperceived externality. A few possibilities are depicted in figure 1.2.

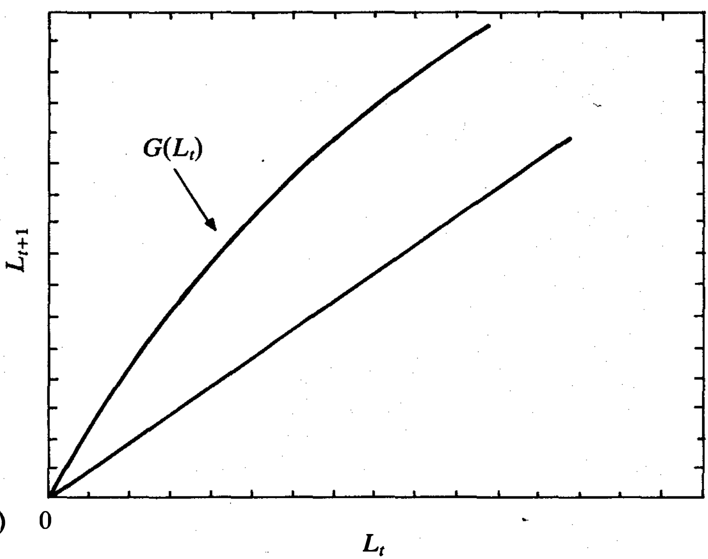
Suppose $G(L_t)$ is a concave function which is zero at $L_t = 0$ and satisfies the Inada condition. Then, in the long run, population will be stationary and per capita income will be constant as in the standard neoclassical growth model. This is shown in figure 1.2(a). Now suppose that $A(L_t)$ is such that $G(L_t)$ is concave and $G'(L_t)$ is bounded away from 1. In this case, we have long-run growth in L_t and hence in per capita income. This is shown in figure 1.2(b).

Suppose now that $A(L_t)$ is a logistic function with a positive asymptote, such as $A(L) = \gamma \exp[-(L - \bar{L})^2/2]$, for $L \geqslant 0$. It can be shown (Raut and Srinivasan, 1991; see also figure 1.2(c)) that there are multiple steady states. Let us denote the nontrivial steady states as L^* and L^{**} (see figure 1.2(c)). Let $\bar{L}$ be the maximum of $G(L_t)$. The local dynamic properties of these steady states depend on the parameter values and the position of $\bar{L}$ relative to L^{**} plays a crucial role in the local dynamics. If the maximum $\bar{L}$ is to the right of L^{**}, then L^{**} is locally stable and there exists a neighborhood around L^{**} within which the system is monotonic. On the other hand, if $\bar{L}$ is to the left of L^{**}, there can be a nongeneric set of parameter values for which the system will exhibit endogenous fluctuations that can be damped, exploding or even chaotic. However, if α is partly influenced by the government through social security schemes, since α can affect γ, the government can shift $\bar{L}$ to the right of L^{**} and thus, locally at least, a social security program can stabilize fluctuations.

We considered more general childrearing costs (Raut and Srinivasan, 1991, section 4a) involving parent's time and depending on the rate of technological change. Naturally these led to more complicated dynamical problems. We show that there could be super exponential growth in per capita income in the long run in the case of some specific functional forms for general costs of childrearing.



(a)



(b)

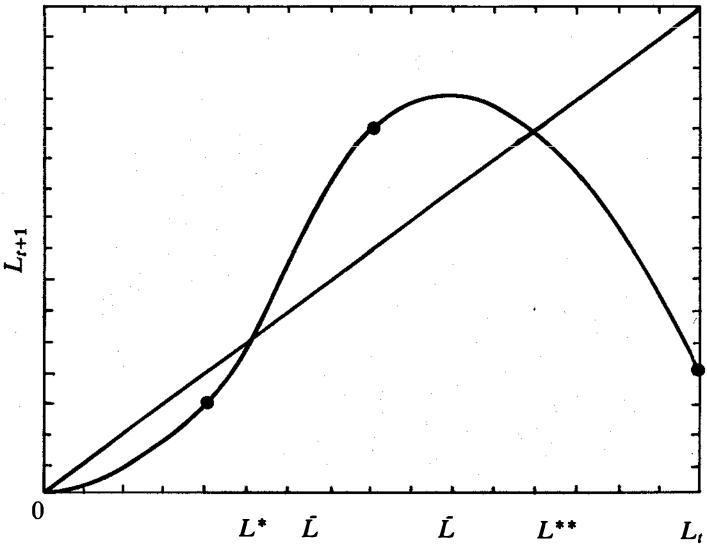


Figure 1.2 (a) Stationary population and income; (b) sustained growth in population and income; (c) complex dynamics of population and income.

5 Conclusions

The starting point of some, though not all, of the recent contributions to growth theory is a misleading characterization of the neo-classical growth theory of the 1960s and earlier as implying that a steady state growth path always exists along which output grows at a rate equal to the exogenously specified rate of growth of the labor force in efficiency units. Thus in the absence of labor-augmenting technical progress, per capita income does not grow along the steady state path. Policies that affect savings (investment) rates have only transient effects on the growth rate of per capita output although its steady state *level* is affected. Even a cursory reading of the literature is enough to convince a reader that neo-classical growth theorists were fully aware that a steady state need not exist and per capita output can grow indefinitely even in the absence of technical progress provided that the marginal product of capital is bounded away from zero by a sufficiently high positive number. Moreover, they showed that, once one departs from the assumption that the marginal product of capital *monotonically* declines to zero as the capital–labor ratio increases indefinitely, multiple steady state growth paths are likely (only some of which are stable) and that the steady state to which a transition path converges depends on initial conditions. Attempts at endogenizing technical progress were also made by theorists of the era.

We argue that the perceived problems of neoclassical growth theory are not inherent features of all the growth models of the era but only of those which assumed the marginal product of capital (or more generally of any reproducible factor) diminishes to *zero* as the input of capital (or that factor) is increased indefinitely relative to other inputs. Instead of directly relaxing this assumption about production technology the "new" growth theorists in effect make assumptions that are analogous to assuming that the marginal product of capital is bounded away from zero. In some of the models this is achieved by introducing a factor other than physical capital (e.g. human capital, stock of knowledge) which is not subject to inexorable returns. In doing so, some authors end up with an aggregate production function that exhibits increasing scale economies. Unsurprisingly in such models multiple equilibria are possible.

We present a model that takes a different approach to endogenizing technical progress and growth by assuming fertility and savings to be *endogenous* and that the size of the total population has an external effect (of a Hicks-neutral type) through either the negative influence of congestion or a positive stimulation of faster innovation. Our model generates a rich set of growth paths of per capita income and consumption, some of which do not converge to a steady state and are even chaotic.

Although the recent revival of growth theory does not constitute as much of a radical departure from its earlier roots as is sometimes thought, it contains a number of innovations, both theoretical and empirical. Further, by reviving policy interest in growth and development problems, the participants in the revival have performed a very useful service to the profession.

Notes

Dedicated to the memory of Sukhamoy Chakravarty whose premature death deprived the world of a profound scholar and India of a dedicated planner. From his earliest publication (1957) Chakravarty contributed significantly to the theoretical and empirical literature on economic growth and planning. He was one of the first (Chakravarty, 1962) among the theorists to raise deep issues of the existence of an optimal growth path. We thank John Conlisk, Isaac Ehrlick, Elhanan Helpman, Robert Lucas Jr, Mukul Majumdar, Tapan Mitra, Assaf Razin, Nouriel Roubini, Xavier Sala-i-Martin, and Robert Solow for their valuable comments on an earlier draft. We apologize to each of them for not necessarily incorporating all their suggestions in the revision and they certainly are not responsible for any errors that still remain.

1 However in Romer (1990) innovation is driven by profit-maximizing entrepreneurs.

2 One can easily prove this as follows. Suppose

$$\inf_{(K,L)>0} \frac{\partial F}{\partial K} \equiv \gamma > 0$$

Since F is homogeneous of degree one, $F(1, L/K) = \partial F/\partial K + (L/K)\partial F/\partial L \geqslant \partial F/\partial K > \gamma > 0$. Now suppose $L \rightarrow 0$; then it follows that $F(1, 0) > 0$.

3 With respect to an appropriate topology in infinite-dimensional space.

4 $\bar{k}^*$ satisfying this equation is called the modified golden rule capital–labor ratio.

5 When such a relationship holds for all t, we say that the economy is on a balanced growth path.

6 We thank Robert Solow and Xavier Sala-i-Martin for pointing this out to us.

7 However, Benhabib and Jovanovic (1991) do not find any evidence for spill-over using US macro data.

8 There are a number of models in the literature in which the interaction of endogenous fertility and productive investment in human capital are analyzed in a growth context. Our purpose is not to survey this literature either. We refer the interested reader to one very interesting such model by Becker et al. (1990).

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