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Capital accumulation, income distribution and endogenous fertility in an overlapping generations general equilibrium model*

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Abstract: This paper studies the intertemporal relationships among population growth, income distribution, inter-generational social mobility, skill composition of the labor force, and household income in an overlapping generations general equilibrium model that aggregates household decisions regarding fertility, savings and investment in human capital of children. It shows that as a consequence of endogenous fertility, the equilibrium path attains steady state from the second generation. Income tax transfer, child taxation, and social security taxation policies that can be devised to affect these variables are also analyzed. The model provides a structural explanation for the inverse household income–child quantity and negative child quality–quantity relationships that are observed in developing countries. It also shows that group interests may hinder the emergence of perfect capital markets with private initiatives.

At some stage in the development of civilization, it must have occurred to some man of unusual forethought that he could, while his children are still young, produce in them a state of mind which will lead them to keep him alive in old age.

...Bertrand Russell in *Power*.

1. Introduction

This paper examines the relationships among population growth, income distribution, quality composition of the labor force, and household income in an overlapping generations general equilibrium model. The model is based on individual decisions regarding savings, fertility and investment in human capital of children, and it integrates an income distribution theory with the

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general equilibrium theory. This provides a unified analytical framework to study the dynamic properties and mutual interdependence of the relationships between number of children and their 'quality' in households, between number of children and household income, fertility differential and income inequality among households, and between investment in human capital of children and inter-generational social mobility, and also implications of these relationships on household savings, on the growth rates of population, aggregate capital stock, and national income, and on the pattern of income distribution over time.

Empirical studies on these linkages are inconclusive. The hypothesis that a faster population growth rate leads to a higher dependency ratio and hence to a lower saving rate has been confirmed by many studies [Leff (1969), and for other references see Raut (1985)], but refuted by others [see Ram (1982)]. Some studies suggest that higher population growth is related to higher income inequality [Adelman and Morris (1973), Ahluwalia (1976), and Chenery et al. (1974)], but others find no significant relationship [Rodgers (1983), and Lam (1984)]. Most studies on developed countries find that almost perfect inter-generational social mobility is attained within three generations [Becker and Tomes (1986)]. Although there are few studies available on the issue for developing countries, Birdsall's (1980) analysis of cross-country income distribution data, Heckman and Hotz's (1986) analysis of Panama's micro data suggest that there is very little inter-generational social mobility in developing countries. Perhaps the only universally accepted findings are the negative relationships between number of children and their quality, and between number of children and household income [Becker (1981, Chapter 5), World Development Report (1984, 69–70), and Birdsall (1980)].

Until recently, theoretical explanations for these relationships were founded on utility maximizing parental behavior in static partial equilibrium frameworks. The quality–quantity and income–quantity relationships have been derived from time allocation, household production, and quality–quantity interaction frameworks [Becker (1965), Willis (1974), and Becker and Lewis (1973)]. These models take expenditures on children as a measure of child quality, and assume that parents invest equally in each child. The above negative relationships between quality and quantity of children, income and quantity of children are then explained by imposing restrictions on the utility functions.[1] Incorporating various types of lucks, the same approach has also been used to explain the observed pattern of inter-generational social mobility in developed countries [Becker and Tomes (1979, 1986)]. Behrman, Pollak and Taubman (1982) provide an alternative

[1]See Rosenzweig and Wolpin (1980), Pollak and Watcher (1975), and Arthur (1982) for a criticism.

theory of allocation of bequests and schooling investments among children based on parental preference towards inequality in children's earnings. These models do not integrate the microeconomics of savings and fertility decisions with the macroeconomics of population growth and capital accumulation. In other words, these models do not take into account the general equilibrium effect of fertility, savings, and human capital investment decisions on the interest rate and the earnings of different skills and feed-back on these decisions. Macro-models have assumed the population growth rate to be either exogenously given or a function of national income, ignoring the micro-economics of household fertility decisions. As a result, policy prescriptions based on these models may not be compatible with individual incentives and hence, ineffective.

Understanding the link between macroeconomic effects and causes of population growth is essential in addressing many policy questions. Can a reduction in income inequality in a society reduce its population growth? Or does rapid population growth lead to increasing income inequality? Do policies of subsidizing investment in child quality such as those of China, Taiwan, Korea, and India reduce or have they already reduced fertility rates significantly? Could improvements in financial markets in rural areas reduce fertility by providing alternatives to investment in children?

Models that endogenize fertility decisions are one-sector growth frameworks where parents are assumed to be altruistic towards all future generations [Becker and Barro (1988), Kemp, Leonard and Long (1984), Nerlove, Razin and Sadka (1987), and Razin and Ben-Zion (1975)]. Because there is only one sector in these growth models, however, they do not lend themselves to the study of income distribution and inter-generational social mobility effects.

The point of departure of our model is the Samuelson (1958)–Diamond (1965) overlapping generations framework; the model extends the works of Nehar (1971), Willis (1980), Eckstein and Wolpin (1985). Individual utility maximizing decisions regarding savings, fertility and investment in human capital of children and the competitive factor rewards constitute the endogenous dynamics of population growth, capital accumulation, income distribution among skill groups and inter-generational social mobility within the general equilibrium framework. The underlying assumptions are that in the absence of well developed capital markets the parents depend on their childeren for old age support. Thus children not only provide utility to parents analogous to the latter's current consumption of goods but more importantly serve as channels of investment for the provision of their old age consumption.[2] Or in other words, the parents view the number of children

[2]This assumption finds support in a number of empirical studies on developing countries. See for instance, Entwisle and Winegarden (1984), Gillaspy and Nugent (1983), World Development Report (1984, 51–52), and Caldwell (1982).

and allocation of income toward their education as investment decisions. I also assume for simplicity of exposition that there are two skill groups (skilled and unskilled) and that costs of investment in human capital, and physical capital vary across skill groups.

In section 2, I set up the model and explain the terminology. In section 3, I prove the existence of a liquidity constrained equilibrium that is defined in section 2. In section 4, I derive the conditions that generate a complete specialization of activities by the two groups, namely that the unskilled parents would invest only in unskilled children and the skilled parents would invest in skilled children and physical capital. In section 5, I study the dynamic nature of the equilibrium rates of interest, income distribution and the growth in national income, population of the two groups, and aggregate capital for Cobb–Douglas economies. In section 6, I consider another economy and compare the results obtained for the Cobb–Douglas economy. In section 7, I study the gains and losses of the two groups and other dynamic effects of introducing perfect capital markets in the Cobb–Douglas economy and compare the results with section 5. In section 8, I study the macro consequence of income redistribution, child-taxation, and introduction of a social security program. In section 9, I summarize the results and conclude the paper.

2. The basic model

The economy consists of an aggregative productive sector and overlapping generations of two types of households. Using capital, skilled labor and unskilled labor, the production sector in each period produces a good that could be consumed or invested. Capital is irreversible, i.e., once invested, it cannot be consumed. Denote by K_t, L_t^S, and L_t^U respectively the aggregate stock of capital, skilled labor, and unskilled labor available for production at time t. Let

$$\boldsymbol{P}=(p_t, q_t, W_t^S, W_t^U)_0^\infty,$$

where p_t, q_t, W_t^S, W_t^U represent respectively the present value of a unit of consumer good, capital good, skilled labor, and unskilled labor available in period t, for $t \geqq 0$, with the numeraire $p_0=1$. Let $1+r_t=q_t/p_t$, the rate of return of capital in period t in terms of the consumer good of the same period.

Assume for simplicity that capital lasts for one period and has zero scrap value and the production function $F(K_t, L_t^S, L_t^U)$ is time invariant.

The *producer's problem* at time t is for given $\boldsymbol{P}$ to choose non-negative K_t, L_t^S, and L_t^U so as to maximize profit

$$p_t F(K_t, L_t^{\mathrm{S}}, L_t^{\mathrm{U}}) - p_t K_t - W_t^{\mathrm{S}} L_t^{\mathrm{S}} - W_t^{\mathrm{U}} L_t^{\mathrm{U}}. \tag{2.1}$$

A solution of this maximization problem yields demands for capital, K_t^{d}, skilled labor, L_t^{Sd}, and unskilled labor, L_t^{Ud} and the supply of total output $Y_t^{\mathrm{s}} = F(K_t, L_t^{\mathrm{S}}, L_t^{\mathrm{U}})$ in period t. The superscripts d and s denote respectively the market demand and supply. Denote the maximized profit by $\Pi_t(\boldsymbol{P})$. Note that for a give $\boldsymbol{P}$ the optimal solution to (2.1) may not be unique, as for instance in the case of constant returns to scale production functions. Therefore, for given $\boldsymbol{P}$, the set of factor demands in period t, $t \geqq 0$,

$$\Lambda_t^F(\boldsymbol{P}) = \{(K_t^{\mathrm{d}}, L_t^{\mathrm{Sd}}, L_t^{\mathrm{Ud}}) \,|\, (2.1) \text{ is maximized}\}$$

is a correspondence. If the markets are perfectly competitive and the production function is homogeneous of degree 1, then $\Pi_t(\boldsymbol{P}) = 0$ for all $t \geqq 0$.

2.1. Household sector

Two types of households are identified with the skill levels of the adults of the household. Sometimes I will refer to skilled parents as rich and unskilled parents as poor. Assume that the individuals belonging to the same group are identical. Each person lives through three periods – young, adult, and old. One unit of labor is supplied inelastically by each member of the adults of that generation. No distinction is made between genders. A young individual is totally dependent on his parent. An adult enjoys parenthood and participates in the labor market to support his family. Let W_t^g be the wage rate in period t of an adult of skill type g. Assume that he gives a constant (over time and across groups)[3] fraction of his income, αW_t^g to his retired parent. Out of his remaining income, he decides on his consumption C_t^{g1}, investment on physical capital s_t^g, number of skilled children $n_t^{g\mathrm{S}}$ and number of unskilled children $n_t^{g\mathrm{U}}$ for $g = \mathrm{S}, \mathrm{U}$, and $t \geqq 0$. I assume that the gestation period of capital is one period and capital is owned by the old each holding shares proportional to his capital contribution. An old person in period t, receives remittances from his children, returns from his investment and profit from his firm, and from these he finances his consumption, C_t^{g2}. To be consistent with the economic realities of the less developed countries, I assume that the agents are liquidity constrained. In section 7, I will examine the implication of this assumption.

Assume that the cost in units of consumer goods of formation of each unit of capital, skilled labor and unskilled labor to an unskilled parent is

[3]The assumption that α is constant across groups and over time is not essential for most results. However, the exogeneity of α is assumed to simplify analysis. I discuss the endogenous determination of α later in this section.

$$\theta_K^{\mathrm{U}}=1+c,\quad \theta_{\mathrm{S}}^{\mathrm{U}}=d_{\mathrm{S}}+f,\quad \theta_{\mathrm{U}}^{\mathrm{U}}=d_{\mathrm{U}}, \tag{2.2}$$

and to a skilled parent is

$$\theta_K^{\mathrm{S}}=1,\quad \theta_{\mathrm{S}}^{\mathrm{S}}=d_{\mathrm{S}},\quad \theta_{\mathrm{U}}^{\mathrm{S}}=d_{\mathrm{U}}+g, \tag{2.3}$$

where, d_{S}, d_{U}, c, f, and g are all positive and constant over time, and $d_{\mathrm{S}}>d_{\mathrm{U}}$.

While several explanations for the above differential costs are plausible, let me point out two of them here. *First*, in many developing countries the poor live in rural areas and urban slums at some distance from schools and colleges, in particular from the good ones. I assume that only low quality schools are located in areas populated by the poor. I interpret $f>0$ as the transport cost a poor parent has to bear in order to send his child to a good school away from home. Similarly, $g>0$ is the transport cost a rich parent has to bear in order to send his child to a lower quality school away from home. $c>0$ can be thought of as a broker's fee a poor parent has to pay in order to obtain technical information about the capital market. A *second* conceivable motivation for these differential costs concerns differences in attitudes towards risk bearing. The production processes of skilled labor and physical capital involve higher risk than that of unskilled labor. Poor parents are probably more risk averse than the rich. Since I did not introduce uncertainty in the model explicitly, I assume that the poor would behave exactly like the rich in a certainty equivalent way, had they received subsidies c and f per unit of capital and skill they form. Furthermore, $g>0$ could be interpreted as psychological compensation needed by a rich parent who suffers status loss if he sends his children to a lower quality school. With this interpretation, the location of school does not matter.

Since the adults can observe the consumption of their old parents and since most people have filial piety, an appropriate utility function for an adult of group g, and generation t would be of the form $U_t^g(C_t^{g2}, C_t^{g1}, C_{t+1}^{g2})$. This will allow the parameter α to be determined by the agents. However, this creates many analytical problems which will be discussed later. For analytical tractability, I consider only the life-cycle utility functions $U_t^g(C_t^{g1}, C_{t+1}^{g2})$ and treat α as exogenously given.

A *representative parent of group g and generation t* chooses a non-negative

$$a_t^g=(C_t^{g1}, C_{t+1}^{g2}, s_t^g, n_t^{g\mathrm{S}}, n_t^{g\mathrm{U}})$$

vector to maximize $U_t^g(C_t^{g1}, C_{t+1}^{g2})$ subject to

$$p_t C_t^{g1}=(1-\alpha)W_t^g-p_t(\theta_K^g s_t^g-\theta_{\mathrm{S}}^g n_t^{g\mathrm{S}}-\theta_{\mathrm{U}}^g n_t^{g\mathrm{U}}), \tag{2.4a}$$

$$p_{t+1}C_{t+1}^{g2}=q_{t+1}s_t^g+\alpha(W_{t+1}^{\mathrm{S}}n_t^{g\mathrm{S}}+W_{t+1}^{\mathrm{U}}n_t^{g\mathrm{U}})+\gamma_{t+1}^g\Pi_{t+1}, \tag{2.4b}$$

where, $\gamma^g_{t+1}=s^g_t/\sum_g L^g_t s^g_t$, the share of the firm he owns. Note that for given $\boldsymbol{P}$ there need not be a unique solution to the above problem. Denote the set of solutions of the above problem by

$$\Lambda^g_t(\boldsymbol{P})=\{a^g_t\in\mathbb{R}^5_+ \mid a^g_t \text{ solves the above problem}\},\quad g=S, \text{ and } U.$$

Note that solution to problem (2.4) determines the supply of capital and two types of labor in period $t+1$ and the consumption demand of adults in period t and of old parents in period $t+1$. Using these solutions $\{a^g_t\}^\infty_0$, and given the *initial conditions* L^g_{-1}, s^g_{-1}, n^{gS}_{-1}, n^{gU}_{-1}, for $g=$ S and U the aggregate supplies of capital, labor and the demands for consumption goods, and investment in human capital are defined recursively as follows:

$$L^{gs}_{-1}\equiv L^g_{-1};\quad C^{g2}_0=q_0 s^g_{-1}+\alpha(W^S_0 n^{gS}_{-1}+W^U_0 n^{gU}_{-1})+\gamma^g_0\Pi_0,$$

$$L^{Ss}_t=\sum_g L^{gs}_{t-1}n^{gS}_{t-1};\quad L^{Us}_t=\sum_g L^{gs}_{t-1}n^{gU}_{t-1};\quad K^s_t=\sum_g L^{gs}_{t-1}s^g_{t-1},$$

$$C^d_t=\sum_g(L^{gs}_t C^{g1}_t+L^{gs}_{t-1}C^{g2}_t), \tag{2.5}$$

$$C^s_t=Y^s_t-\sum_g L^{gs}_t(\theta^g_K s^g_t-\theta^g_S n^{gS}_t-\theta^g_U n^{gU}_t)$$

$$=\text{total income}-\text{total investment in period } t.$$

We can now define the excess supplies of each commodity in different periods:

$$\eta^K_t=K^s_t-K^d_t;\quad \eta^g_t=L^{gs}_t-L^{gd}_t;\quad \eta^C_t=C^s_t-c^d_t,\quad g=\text{S, U, and } t\geqq 0 \tag{2.6}$$

Definition 2.1. A *perfect foresight liquidity constrained equilibrium* is a non-negative price vector

$$\boldsymbol{P}=(p_t,q_t,W^S_t,W^U_t)^\infty_0,$$

and

$$a^g_t=(C^{g1}_t,C^{g2}_{t+1},s^g_t,n^{gS}_t,n^{gU}_t)\in\Lambda^g_t(\boldsymbol{P}),$$

and

$$(K^d_t,L^{Sd}_t,L^{Ud}_t)\in\Lambda^F_t(\boldsymbol{P}),\quad g=\text{S and U},\quad t\geqq 0,$$

such that for all $t \geqq 0$, $\Pi_t(\boldsymbol{P})=0$ and $\eta_t^K=\eta_t^S=\eta_t^U=\eta_t^C=0$; and if there is an excess supply of any commodity in any period its price is zero.

Perfect foresight equilibrium obtains because all agents are assumed to know the model. So they can use the model to predict the behavior of other agents and thus the price vector $\boldsymbol{P}$. Because of the assumption that α is exogenously given, the perfect foresight assumption requires the parents of generation t only to compute the q_{t+1}, W_{t+1}^S, and W_{t+1}^U, provided that such prices are unique. This uniqueness is very much essential for the working of the economy otherwise there will exist coordination problems in the sense that if agents form different price expectations, there is no reason for the markets to clear.

If instead we use $U_t^g(C_t^{g2}, C_t^{g1}, C_{t+1}^{g2})$, and make α endogenous, observe that the amount, α_t^g, a g-type adult of the tth generation transfers to his parents will depend on α_{t+1}^g, for $g=\mathrm{S}$ and U, the computation of which in turn requires the knowledge of α_{t+2}^g, for $g=\mathrm{S}$ and U, and so on. Unlike in our model, here parents have to possess tremendous amounts of computational ability to do this. Moreover, the uniqueness of α_t^g's may be difficult to guarantee. Furthermore, this is a dynamic programming problem to which there is no analytical solution.

3. Existence theorem

The following assumptions are made to prove the existence of equilibrium.

Assumption 1. F exhibits constant returns to scale, F has all continuous partial derivatives, and $F(x,y,z)=0$ if $x \cdot y \cdot z=0$, i.e., all three inputs are necessary for production.

Assumption 2 (Inada condition). For all $x>0$,

$$\lim_{z \to x} U_{t1}^g(z,x) = \lim_{z \to \infty} U_{t2}^g(x,z) = \infty,$$

where U_{t1}^g and U_{t2}^g are the partial derivatives of $U_t^g(z,x)$ with respect to z and x, respectively.

Assumption 3. $U_t^g(z,x)$ is strictly quasi-concave and monotonic for $g=\mathrm{S,U}$, and $t \geqq 0$.

Theorem 3.1. Under Assumptions 1–3, there exists a perfect foresight liquidity constrained equilibrium.

Proof. See appendix.

The following proposition will be used in proving Theorem 3.1.

Proposition 3.2 (Walras' Law). *For any $t \geqq 0$, positive price vector*

$$\boldsymbol{P}=(p_t, q_t, W_t^{\mathrm{S}}, W_t^{\mathrm{U}})_0^{\infty}, \quad a_t^g=(C_t^{g1}, C_{t+1}^{g2}, s_t^g, n_t^{g\mathrm{S}}, n_t^{g\mathrm{U}}) \in \Lambda_t^g(\boldsymbol{P}),$$

and

$$(K_t^{\mathrm{d}}, L_t^{\mathrm{Sd}}, L_t^{\mathrm{Ud}}) \in \Lambda_t^{\mathrm{F}}(\boldsymbol{P}), \qquad g=\mathrm{S} \text{ and } U, \quad t \geqq 0,$$

$$\eta_t^K=\eta_t^{\mathrm{S}}=\eta_t^{\mathrm{U}}=0 \Rightarrow \eta_t^{\mathrm{C}}=0.$$

Proof. See appendix.

Remark 3.3. The proof of the above theorem holds even if the fraction α of income transferred from an adult to his retired parent varies over generations and across groups, so long as it is fixed and known to the decision makers.

4. Conditions for complete specialization

I now derive the conditions under which the general equilibrium allocations will produce $s_t^{\mathrm{U}}=n_t^{\mathrm{US}}=n_t^{\mathrm{SU}}=0$ and s_t^{S}, n_t^{SS}, $n_t^{\mathrm{UU}}>0$, which are observed in less developed countries. Such an allocation will be termed as *complete specialization*. Note that I can normalize the equilibrium prices such that $p_t=1$ for all $t \geqq 1$.

The Kuhn–Tucker conditions for parents' optimization problems are

$$1+r_{t+1}/\theta_{\mathrm{K}}^g \leqq U_{t1}^g/U_{t2}^g \qquad (=\text{if } s_t^g>0), \tag{4.1}$$

$$\alpha W_{t+1}^{\mathrm{S}}/\theta_{\mathrm{S}}^g \leqq U_{t1}^g/U_{t2}^g \qquad (=\text{if } n_t^{g\mathrm{S}}>0), \tag{4.2}$$

$$\alpha W_{t+1}^{\mathrm{U}}/\theta_{\mathrm{U}}^g \leqq U_{t1}^g/U_{t2}^g \qquad (=\text{if } n_t^{g\mathrm{U}}>0), \tag{4.3}$$

where U_{t1}^g and U_{t2}^g are the partial derivatives of U_t^g with respect to C_t^{g1} and C_t^{g2}, respectively.

First-order conditions of the skilled parents imply that for $s_t^{\mathrm{S}}, n_t^{\mathrm{SS}}>0$,

$$\alpha W_{t+1}^{\mathrm{S}}/d_{\mathrm{S}}=1+r_{t+1}=U_{t1}^{\mathrm{S}}/U_{t2}^{\mathrm{S}}, \tag{4.4}$$

and for $n_t^{\mathrm{SU}}=0$,

$$\alpha W^{\mathrm{S}}_{t+1}/d_{\mathrm{S}} > \alpha W^{\mathrm{U}}_{t+1}/d_{\mathrm{U}} + g. \tag{4.5}$$

Similarly from the first-order conditions of an unskilled parent's problem, the condition for $s^{\mathrm{S}}_t = n^{\mathrm{US}}_t = 0$ is

$$\frac{\alpha W^{\mathrm{U}}_{t+1}}{d_{\mathrm{U}}} > \max\left\{\frac{\alpha W^{\mathrm{S}}_{t+1}}{d_{\mathrm{S}}+f}, \frac{1+r_{t+1}}{1+c}\right\}. \tag{4.6}$$

Note that if any positive consumption in either period of one's life is infinitely valued over no consumption (i.e., if U^g_t satisfies Assumption 2), then adults in both groups will save a positive amount. An optimal portfolio allocation of these savings among physical capital, number of unskilled children and skilled children will depend on their respective rates of returns. Under conditions (4.4)–(4.6), observe that unskilled parents will find it most productive to invest only on unskilled children and the skilled parents only on capital and skilled children. That is, these conditions are necessary and sufficient for a complete specialization. Thus we have proved:

Proposition 4.1. The perfect foresight liquidity constrained equilibrium will result in a complete specialization in the sense that $s^{\mathrm{U}}_t = n^{\mathrm{US}}_t = n^{\mathrm{SU}}_t = 0$ *and* s^{S}_t, n^{SS}_t, $n^{\mathrm{UU}}_t > 0$, if and only if *(4.4)–(4.6) are satisfied.*

We do not know whether the skilled labor earns higher wages than the unskilled labor. It is, however, clear that the completely specialized economies will have no inter-generational social mobility. Although (4.4)–(4.6) provide conditions for a complete specialization, they involve equilibrium prices. What are the bounds on c, f, g in terms of the parameters of the production function, and utility functions that will generate completely specialized equilibrium? What are the fertility levels of the two groups? Are the rates of returns of the two groups equalized in equilibrium? If not who gains and who loses from imperfections in capital markets? What are the long-run properties of a completely specialized equilibrium? Is an equilibrium unique?

For general production and utility functions, all we can say about the above issues is that the wage differential is bounded by

$$\frac{d_{\mathrm{S}}+f}{d_{\mathrm{U}}} > \frac{W^{\mathrm{S}}_{t+1}}{W^{\mathrm{U}}_{t+1}} > \frac{d_{\mathrm{S}}}{d_{\mathrm{U}}+g} \quad \text{for} \quad \text{all } t \geqq 0, \tag{4.7}$$

which follows from (4.5) and (4.6). To shed light on other issues I consider specific production functions and utility functions.

From now on I will consider only the completely specialized economies, for convenience of notation I will denote s_t^{S}, n_t^{SS}, and n_t^{UU} respectively by s_t, n_t^{S}, and n_t^{U}.

5. Cobb–Douglas economy

Assume the following Cobb–Douglas production function:

$$F(K_t, L_t^{\mathrm{S}}, L_t^{\mathrm{U}}) = (K_t)^{\sigma_1}(L_t^{\mathrm{S}})^{\sigma_2}(L_t^{\mathrm{U}})^{\sigma_3}, \qquad \sigma_1+\sigma_2+\sigma_3=1. \tag{5.1}$$

Utility functions are

$$U_t^g(C_t^{g1}, C_{t+1}^{g2}) = C_t^{g1}\cdot C_{t+1}^{g2} \quad \text{for} \quad g=\mathrm{S},\mathrm{U}, \quad t\geqq 0. \tag{5.2}$$

Note that perfect competition in factor markets implies

$$1+r_{t+1}=\sigma_1 F_{t+1}/K_{t+1}, \quad W_{t+1}^{\mathrm{S}}=\sigma_2 F_{t+1}/L_{t+1}^{\mathrm{S}}, \quad W_{t+1}^{\mathrm{U}}=\sigma_3 F_{t+1}/L_{t+1}^{\mathrm{U}}, \tag{5.3}$$

where F_{t+1} is the total output in period $t+1$. Substituting this in (4.4) and noting that $L_{t+1}^{\mathrm{S}}=L_t^{\mathrm{S}}n_t^{\mathrm{S}}$, and $K_{t+1}=L_t^{\mathrm{S}}s_t$, we have

$$s_t=\eta x_t^{\mathrm{S}}, \quad \text{where} \quad \eta=\sigma_1 d_{\mathrm{S}}/\alpha\sigma_2. \tag{5.4}$$

Note that in a completely specialized economy, an unskilled parent's decision problem reduces to choosing $n_t^{\mathrm{U}}>0$. The first-order condition for his utility maximization yields

$$n_t^{\mathrm{U}}=\beta^{\mathrm{U}}W_t^{\mathrm{U}}, \quad \text{where} \quad \beta^{\mathrm{U}}=(1-\alpha)/2d_{\mathrm{U}}. \tag{5.5}$$

Eliminating s_t from the budget constraints using (5.4), the skilled parent's problem reduces to determining $n_t^{\mathrm{S}}>0$. The first-order condition for his utility maximization yields

$$n_t^{\mathrm{S}}=\beta^{\mathrm{S}}W_t^{\mathrm{S}}, \quad \text{where} \quad \beta^{\mathrm{S}}=\alpha(1-\alpha)\sigma_2/[2d_{\mathrm{S}}(\sigma_1+\alpha\sigma_2)]. \tag{5.6}$$

Now (5.4) and (5.6) imply

$$s_t=\beta^K W_t^{\mathrm{S}}, \quad \text{where} \quad \beta^K=(1-\alpha)\sigma_1/[2(\sigma_1+\alpha\sigma_2)]. \tag{5.7}$$

Note that the equilibrium solution given by (5.5)–(5.7) is unique.

To derive the conditions for a complete specialization, substitute (5.3) in (4.7) and get

$$\frac{d_{\mathrm{U}}}{d_{\mathrm{S}}+f}<\frac{\sigma_3 L_{t+1}^{\mathrm{S}}}{\sigma_2 L_{t+1}^{\mathrm{U}}}=\frac{\sigma_3 L_t^{\mathrm{S}} n_t^{\mathrm{S}}}{\sigma_2 L_t^{\mathrm{U}} n_t^{\mathrm{U}}}=\frac{\sigma_3 L_t^{\mathrm{S}} W_t^{\mathrm{S}} \beta^{\mathrm{S}}}{\sigma_2 L_t^{\mathrm{U}} W_t^{\mathrm{U}} \beta^{\mathrm{U}}}=\frac{\sigma_3 \sigma_2 F_t \beta^{\mathrm{S}}}{\sigma_2 \sigma_3 F_t \beta^{\mathrm{U}}} \Rightarrow \frac{f}{d_{\mathrm{S}}}>\frac{\sigma_1}{\alpha \sigma_2}.$$

Proceeding similarly for the other inequality in (4.7) and (4.5), we can prove the following proposition:

Proposition 5.1. If c, f, g, d_{S}, and d_{U} satisfy

$$f/d_{\mathrm{S}}>\sigma_1/\alpha\sigma_2, \quad g>\sigma_1/(\sigma_1+\alpha\sigma_2), \quad \text{and } c>\sigma_1/\alpha\sigma_2, \tag{5.8}$$

then the equilibrium will result in a complete specialization.

Let r_{t+1}^{S} and r_{t+1}^{U} be respectively the rates of returns of the skilled and the unskilled parents of generation t from their investments in children. I.e.,

$$1+r_{t+1}^{\mathrm{S}}=\alpha W_{t+1}^{\mathrm{S}}/d_{\mathrm{S}} \text{ and } 1+r_{t+1}^{\mathrm{U}}=\alpha W_{t+1}^{\mathrm{U}}/d_{\mathrm{U}}.$$

Although the skilled parents have comparative advantage in producing skilled children and physical capital, the unskilled parents also have comparative advantage at least in producing unskilled children. Would the equilibrium rates of returns of the two groups be then equalized? I.e., would $r_{t+1}^{\mathrm{S}}=r_{t+1}^{\mathrm{U}}$?

Proposition 5.2. For a Cobb–Douglas economy, there does not exist a perfect foresight liquidity constrained equilibrium with complete specialization such that $r_{t+1}^{\mathrm{S}}\leqq r_{t+1}^{\mathrm{U}}$, for any $t\geqq 0$.

As a corollary to the above proposition, it is clear that in the Cobb–Douglas economy $W_{t+1}^{\mathrm{S}}/W_{t+1}^{\mathrm{U}}>d_{\mathrm{S}}/d_{\mathrm{U}}>1$. Therefore, in this economy, the skilled workers are rich and the unskilled workers are poor.

It follows from (5.5) and (5.6) that

$$\frac{n_t^{\mathrm{U}}}{n_t^{\mathrm{S}}}=\Psi\cdot\frac{W_t^{\mathrm{U}}}{W_t^{\mathrm{S}}}, \quad \text{where} \quad \Psi=\frac{(\sigma_2\alpha+\sigma_1)d_{\mathrm{S}}}{\sigma_2\alpha d_{\mathrm{U}}}>1. \tag{5.9}$$

It is clear from (5.9) that for the same level of wages which are also their total income, the unskilled parents tend to have larger families than the skilled parents. More precisely, up to a certain level of income inequality of the two groups, with $W_t^{\mathrm{U}}/W_t^{\mathrm{S}}>1/\Psi$, the unskilled parents will have larger families than the skilled parents. This supports the commonly observed quality–quantity trade-off and negative income–quantity relationship at the macro level.

It follows from (5.5) and (5.6) that

$$W^{\mathrm{U}}_{t+1}/W^{\mathrm{S}}_{t+1}=\sigma_3/\sigma_2\cdot((L^{\mathrm{S}}_t n^{\mathrm{S}}_t)/(L^{\mathrm{U}}_t n^{\mathrm{U}}_t)). \tag{5.10}$$

By substituting the values of n^{S}_t and n^{U}_t in (5.10) we have

$$\frac{W^{\mathrm{U}}_{t+1}}{W^{\mathrm{S}}_{t+1}}=\frac{\sigma_3\sigma_2\alpha d_{\mathrm{U}}}{\sigma_2(\sigma_2\alpha+\sigma_1)d_{\mathrm{S}}}\cdot\frac{L^{\mathrm{S}}_t}{L^{\mathrm{U}}_t}\cdot\frac{1}{(W^{\mathrm{U}}_t/W^{\mathrm{S}}_t)}. \tag{5.11}$$

Eq. (5.10) implies that a larger family size of the unskilled parents would cause a wider income differential for their own children as compared to the children of the skilled parents. The extent of the inequality would, however, vary inversely with the income disparities between the rich and the poor of their own generation as shown by (5.11). Also note that the income disparities of the children of the two groups would be larger, were the unskilled parents proportionately larger in number than the rich.

I now derive the dynamic equilibrium path of population, aggregate capital, national income, and wages of the two groups. Suppose the initial condition is given by L^{S}_0, L^{U}_0, K_0. Suppose in period $t=0$, the wages of the two groups are exogenously given to be W^{S}_0, and W^{U}_0, and for all $t\geqq 1$, the factor returns are determined competitively. Then using (5.5)–(5.7) one can derive the dynamic path of the economy as follows:

Note that $L^{\mathrm{U}}_{t+1}=L^{\mathrm{U}}_t n^{\mathrm{U}}_t=L^{\mathrm{U}}_t\beta^{\mathrm{U}}W^{\mathrm{U}}_t=\beta^{\mathrm{U}}\sigma_3 F_t$ is valid for $t\geqq 1$ because of our above assumption on factor returns. Taking logarithm on both sides and following the same procedure for the other factors, we get for all $t\geqq 1$,

$$\ln K_{t+1}=\ln(\beta^K\sigma_2)+\sigma_1\ln K_t+\sigma_2\ln L^{\mathrm{S}}_t+\sigma_3\ln L^{\mathrm{U}}_t, \tag{5.12a}$$

$$\ln L^{\mathrm{S}}_{t+1}=\ln(\beta^{\mathrm{S}}\sigma_2)+\sigma_1\ln K_t+\sigma_2\ln L^{\mathrm{S}}_t+\sigma_3\ln L^{\mathrm{U}}_t, \tag{5.12b}$$

$$\ln L^{\mathrm{U}}_{t+1}=\ln(\beta^{\mathrm{U}}\sigma_3)+\sigma_1\ln K_t+\sigma_2\ln L^{\mathrm{S}}_t+\sigma_3\ln L^{\mathrm{U}}_t. \tag{5.12c}$$

An unique solution to this three-dimensional first-order linear difference equation is easy to compute. This is given by (i)–(iii) in the following proposition.

Proposition 5.3. If the wages and rentals are determined competitively from $t\geqq 1$, then the dynamic path of the economy, for $t\geqq 2$, is given by

(*i*) $L^{\mathrm{U}}_t=L^{\mathrm{U}}_2(1+g)^{t-2}$,
(*ii*) $L^{\mathrm{S}}_t=L^{\mathrm{S}}_2(1+g)^{t-2}$,
(*iii*) $K_t=K_2(1+g)^{t-2}$,
(*iv*) $W^{\mathrm{U}}_t=W^{\mathrm{U}}_2$,

(*v*) $W_t^S = W_2^S$,

where $1+g=(\beta^K\sigma_2)^{\sigma_1}(\beta^S\sigma_2)^{\sigma_2}(\beta^U\sigma_3)^{\sigma_3}$, *while for* $t=1$ *and* 2, K_t, L_t^S, L_t^U, W_t^S, W_t^U *could be calculated directly.*

The above proposition implies that the economy converges to a steady state from the second period and the steady state growth rate is independent from the initial conditions. Hence the steady state is unique and globally stable; a transitory shock in any period affects the wage rates and fertility rates only in the first two periods and has no long-run effects. It could be easily shown that the higher are the costs of raising children (i.e., d_S and d_U), or the higher are the transfers from children to parents (i.e. α), the lower will be the steady state growth rates.

6. Two-goods economy

To contrast with some of the results of the previous section, I consider another production function[4] as follows:

$$F(K_t, L_t^S, L_t^U) = A(K_t)^{\sigma}(L_t^S)^{1-\sigma} + L_t^U, \qquad 0<\sigma<1. \tag{6.1}$$

While the skilled labor cannot produce anything without capital, unskilled labor can produce without capital. Neither type of labor is helpful in other type's production condition. The interpretation of the production function (6.1) is as follows. Imagine that land is abundant, and the unskilled persons are agricultural laborers. Suppose that working on land, each unit of unskilled labor can produce one unit of rice. Working in manufacturing, L_t^S skilled labor, and K_t capital can produce $A(K_t)^{\sigma}$ $(L_t^S)^{1-\sigma}$ units of iron. Now suppose that there is an international market where rice could be exchanged for iron, unit for unit. Then F represents the value of output of rice and iron at international prices. I will refer to this economy as a *two-goods economy.*

Proceeding in the same way as in the previous section, note that for a completely specialized economy, $W_t^U=1$, for all $t \geqq 0$. Therefore, $1+r_t^U = (1-\alpha)/d_U$, for $t \geqq 1$. Let $k_{t+1} = K_{t+1}/L_{t+1}^S$. Then (4.4) reduces to

$$g'(k_{t+1}) = (\alpha/d_S)\cdot(g(k_{t+1}) - k_{t+1}g'(k_{t+1})) \quad \text{for} \quad t \geqq 0, \tag{6.2}$$

where $g(k)=Ak^{\sigma}$. From (6.2) it is obvious that while the Cobb–Douglas economy attains steady state from the second generation, this economy will

[4]Many of the following results are also true for a more general production function, $F(K_t, L_t^S, L_t^U) = G(K_t, L_t^S) + \rho L_t^U$, where G is a constant returns to scale production function, and $\rho > 0$.

attain steady state from the first generation. The equilibrium solutions are given by

$$n_t^{\mathrm{U}}=\gamma^{\mathrm{U}} W_t^{\mathrm{U}}, \quad \text{where} \quad \gamma^{\mathrm{U}}=\frac{(1-\alpha)}{2d_{\mathrm{U}}}, \tag{6.3}$$

$$n_t^{\mathrm{S}}=\gamma^{\mathrm{S}} W_t^{\mathrm{S}}, \quad \text{where} \quad \gamma^{\mathrm{S}}=\frac{\alpha(1-\alpha)(1-\sigma)}{2d_{\mathrm{S}}(\sigma+\alpha(1-\sigma))}, \tag{6.4}$$

$$s_t=\gamma^{K} W_t^{\mathrm{S}}, \quad \text{where} \quad \gamma^{K}=\frac{(1-\alpha)\sigma}{2(\sigma+\alpha(1-\sigma))}. \tag{6.5}$$

Note that $W_t^{\mathrm{S}}=A(1-\sigma)\tilde{k}^{\sigma}$, and $1+r_t^{\mathrm{S}}=A\sigma\tilde{k}^{1-\sigma}$, where $\tilde{k}=\sigma d_{\mathrm{S}}/(1-\sigma)\alpha$ is the unique[5] solution of (6.2), for $t\geqq 1$. In this economy it is clear that suitable c, g, f, d_{S}, and d_{U} could be chosen such that r_t^{S} can be larger, equal or smaller than r_t^{U}. The following are the conditions for $r_t^{\mathrm{U}}=r_t^{\mathrm{S}}$, for all $t\geqq 1$:

$$A(1-\sigma)^{1-\sigma}\cdot\sigma^{\sigma}\cdot\alpha^{\sigma}=d_{\mathrm{S}}/d_{\mathrm{U}} \tag{6.6}$$

and $c,g,f\geqq 0$ could be arbitrary. For such an economy, note that for $t\geqq 0$,

$$W_{t+1}^{\mathrm{S}}/W_{t+1}^{\mathrm{U}}=A(1-\sigma)(\gamma^{K}/\gamma^{\mathrm{S}})^{\sigma}=d_{\mathrm{S}}/d_{\mathrm{U}}$$

by (6.6). Hence the following proposition is true.

Proposition 6.1. If (6.6) is satisfied, then for all $t\geqq 0$, *(i)* $r_{t+1}^{\mathrm{S}}=r_{t+1}^{\mathrm{U}}$, *(ii)* $n_{t+1}^{\mathrm{U}}>n_{t+1}^{\mathrm{S}}$, *and (iii)* $W_{t+1}^{\mathrm{S}}/W_{t+1}^{\mathrm{U}}=d_{\mathrm{S}}/d_{\mathrm{U}}>1$.

Note that for any c, g, and $f>0$, the above results are true. The proposition implies that at the macro level the negative relationships between quality and quantity of children and between income and quantity of children are only transitory (i.e., true only for first two generations) in the Cobb–Douglas economy, and in the two goods economy these relationships are permanent.

7. Perfect capital markets

In this section, we study who gains and who loses from introducing perfect capital markets. Could the capital markets be made perfect with private initiatives? What are its short-run and long-run effects on the rates of growth

[5]This solution is unique, see Raut (1987).

in population, capital accumulation, national income, and on skill composition of the labor force and income distribution?

Perfect capital markets means that the rates of returns of the two groups are equalized. I.e.,

$$1+r_{t+1}^{\mathrm{U}}=1+r_{t+1}^{\mathrm{S}}=1+r_{t+1} \quad \text{for} \quad t \geqq 0. \tag{7.1}$$

Note that for $c, f, g>0$, the equilibrium will be completely specialized as the unskilled parents' rates of returns from investment in capital and skilled children will be lower than that from unskilled children. Similarly, for the skilled parents, the rates of returns from investing in unskilled children is lower than the other two investments. But for a Cobb–Douglas economy, Proposition 5.2 implies that such an equilibrium cannot exist. In fact, government has to act as a financial intermediary to siphon the savings of the unskilled parents to the capital markets so that the unskilled parents do not have to incur the extra information processing costs, c, for their investment.

From (7.1) it is clear that $W_{t+1}^{\mathrm{S}}/W_{t+1}^{\mathrm{U}}=d_{\mathrm{S}}/d_{\mathrm{U}}$. Under perfect capital markets, skilled labor earns higher wages than unskilled labor. However, income inequality is less than that in the liquidity constraint case.

Note that for a Cobb–Douglas economy (7.1) implies that for $t \geqq 0$,

$$\alpha W_{t+1}^{\mathrm{U}}/d_{\mathrm{U}}=1+r_{t+1} \quad \text{implies} \quad \frac{\alpha\sigma_3 F_{t+1}}{d_{\mathrm{U}} L_t^{\mathrm{U}} n_t^{\mathrm{U}}}=\frac{\sigma_1 F_{t+1}}{K_{t+1}}, \quad \text{implies}$$

$$d_{\mathrm{U}} L_t^{\mathrm{U}} n_t^{\mathrm{U}}=(\alpha\sigma_3/\sigma_1)\cdot K_{t+1}. \tag{7.2}$$

Similarly, from (7.1) we derive for $t \geqq 0$,

$$d_{\mathrm{S}} L_t^{\mathrm{S}} n_t^{\mathrm{S}}=(\alpha\sigma_2/\sigma_1)\cdot K_{t+1}. \tag{7.3}$$

Eqs. (7.1) and (7.2) provide the equilibrium investment in unskilled children and skilled children in period t, respectively. For Cobb–Douglas utility functions total savings (including the part invested on human capital) of the two groups are given by $s_t^{\mathrm{U}}=(1-\alpha) W_t^{\mathrm{U}}/2$ and $s_t^{\mathrm{S}}=(1-\alpha) W_t^{\mathrm{S}}/2$. Therefore in equilibrium,

$K_{t+1}=$ aggregate savings of the two groups $-$ aggregate investments in human capital of two groups

$$=\frac{(1-\alpha)}{2}(L_t^{\mathrm{S}} W_t^{\mathrm{S}}+L_t^{\mathrm{U}} W_t^{\mathrm{U}})-(L_t^{\mathrm{S}} d_{\mathrm{S}} n_t^{\mathrm{S}}+L_t^{\mathrm{U}} d_{\mathrm{U}} n_t^{\mathrm{U}})$$

$$=\frac{(1-\alpha)}{2}(L_t^S W_t^S + L_t^U W_t^U) - \frac{\alpha(\sigma_3+\sigma_2)}{\sigma_1}\cdot K_{t+1},$$

from which we derive that

$$K_{t+1}=\frac{\sigma_1(1-\alpha)}{2(\sigma_1+\alpha(1-\sigma_1))}\cdot(L_t^S W_t^S + L_t^U W_t^U). \tag{7.4}$$

Suppose that the factors are rewarded competitively for $t \geqq 0$, then we can easily derive that

$$L_t^S W_t^S + L_t^U W_t^U = \sigma_2 F_t + \sigma_3 F_t = (1-\sigma_1)F_t. \tag{7.5}$$

Noting that $\sigma_3 F_t / L_t^U = W_t^U$ and $\sigma_2 F_t / L_t^S = W_t^S$, (7.2)–(7.4) imply

$$n_t^{S*}=\frac{\alpha(1-\alpha)(1-\sigma_1)}{2d_S[\sigma_1+\alpha(1-\sigma_1)]}\cdot W_t^S, \tag{7.6}$$

$$n_t^{U*}=\frac{\alpha(1-\alpha)(1-\sigma_1)}{2d_U[\sigma_1+\alpha(1-\sigma_1)]}\cdot W_t^U, \tag{7.7}$$

where * denotes the equilibrium quantities under perfect capital markets assumptions. Comparing this solution with the liquidity constraint solution, we have

$$\frac{n_t^{U*}}{n_t^U}=\frac{\alpha(1-\sigma_1)}{\sigma_1+\alpha(1-\sigma_1)}<1, \tag{7.8}$$

$$\frac{n_t^{S*}}{n_t^S}=\frac{(1-\sigma_1)(\sigma_1+\sigma_2\alpha)}{\sigma_2[\sigma_1+\alpha(1-\sigma_1)]}>1, \tag{7.9}$$

$$\frac{K_{t+1}^*}{K_{t+1}^*}=\frac{(1-\sigma_1)(\sigma_1+\sigma_2\alpha)}{\sigma_2[\sigma_1+\alpha(1-\sigma_1)]}>1. \tag{7.10}$$

One can derive after some simplifications that for all $t \geqq 0$,

$$\frac{F_{t+1}^*}{F_{t+1}}=\frac{(\sigma_1+\alpha\sigma_2)(1-\sigma_1)}{\sigma_2[\sigma_1+\alpha(1-\sigma_1)]}\cdot\left[\frac{\alpha\sigma_2}{\sigma_1+\alpha\sigma}\right]^{\sigma_3}. \tag{7.11}$$

The dynamic path of the equilibrium is characterized by equations similar to (5.12) with appropriate β's and one can derive that

$$\frac{1+g^*}{1+g}=\frac{(\sigma_1+\alpha\sigma_2)(1-\sigma_1)}{\sigma_2[\sigma_1+\alpha(1-\sigma_1)]}\cdot\left[\frac{\alpha\sigma_2}{\sigma_1+\alpha\sigma}\right]^{\sigma_3}, \tag{7.12}$$

where g^* is the steady-state growth rate under perfect capital markets. Eqs. (7.11) and (7.12) will be either greater than or less than one depending on the parameter values. For instance, $\sigma_1=0.5$, $\sigma_2=0.3$, and $\alpha=0.05 \Rightarrow g^*/g<1$; whereas, $\sigma_1=0.5$, $\sigma_2=0.3$, and $\alpha=0.3 \Rightarrow g^*/g>1$. Thus we have proved the following proposition:

Proposition 7.1. In a liquidity constrained equilibrium, making capital markets perfect in period t will result in an increase in aggregate capital, in skilled population, and a decrease in unskilled population in period $t+1$. However, the effect on total output as well as long-run steady-state growth rate will be higher or lower depending on the parameter values.

Observe that since as a result of introducing perfect capital markets both capital and skilled labor increase, unskilled labor falls, r_{t+1}^{U} rises and r_{t+1}^{S} drops. Or in other words, the unskilled parents gain and the skilled parents lose from perfection in capital markets. Although an individual in the skilled group can be benefited from borrowing from the unskilled parents, the skilled group's collective interest may devise mutual sanctions and devices to prohibit this. This indicates that by private initiatives perfect capital markets cannot be instituted.

8. Pareto optimality and income redistribution policies

I consider three income redistribution policies in a Cobb–Douglas economy, namely, a lump sum tax transfer from the rich to the poor, taxation of parents for having unskilled children, and social security taxation. I also show that if the voluntary transfers, α, from adult to old is smaller than a certain value, then the decentralized economy follows a high population growth and low capital accumulation path in Pareto sense as defined below.

Definition 8.1. A *feasible allocation* is

$$(C_t^{g1}, C_{t+1}^{g2}, s_t^g, n_t^{gS}, n_t^{gU})_0^{\infty} \geqq 0, \quad g=\mathrm{S} \text{ and } \mathrm{U},$$

such that

$$C_t + K_{t+1} + J_{t+1} = F(K_t, L_t^{\mathrm{S}}, L_t^{\mathrm{U}}), \quad \text{where}$$

$$L_t^{\mathrm{S}} = \sum_g L_{t-1}^g n_{t-1}^{gS},$$

$$L_t^{\mathrm{U}} = \sum_g L_{t-1}^g n_{t-1}^{g\mathrm{U}},$$

$$K_t = \sum_g L_{t-1}^g s_{t-1}^g,$$

$$C_t = \sum_g (L_t^g C_t^{g1} + L_{t-1}^g C_t^{g2}),$$

$$J_{t+1} = \sum_g L_t^g \cdot [(1-\theta_K^g) s_t^g + \theta_{\mathrm{S}}^g n_t^{g\mathrm{S}} + \theta_{\mathrm{U}}^g n_t^{g\mathrm{U}}],$$

given the initial conditions $K_0, L_0^{\mathrm{S}}, L_0^{\mathrm{U}}$.

Definition 8.2. A feasible allocation

$$(\bar{C}_t^{g1}, \bar{C}_{t+1}^{g2}, \bar{s}_t^g, \bar{n}_t^{g\mathrm{S}}, \bar{n}_t^{g\mathrm{U}})_0^\infty \geqq 0, \quad g = \mathrm{S} \text{ and } \mathrm{U},$$

is *cohort-wise Pareto optimal* if there does not exist another feasible allocation

$$(C_t^{g1}, C_{t+1}^{g2}, s_t^g, n_t^{g\mathrm{S}}, n_t^{g\mathrm{U}})_0^\infty \geqq 0, \quad g = \mathrm{S} \text{ and } \mathrm{U},$$

such that

$$U_t^g(C_t^{g1}, C_{t+1}^{g2}) \geqq U_t^g(\bar{C}_t^{g1}, \bar{C}_{t+1}^{g2}) \quad \text{for} \quad t \geqq 0, \quad \text{and} \quad g = \mathrm{S} \text{ and } \mathrm{U}$$

with at least one strict inequality.

Remark 8.3. This definition of optimality is limited in that it uses consumption of a representative member of each cohort as the criterion for comparing different consumption streams. The number of persons in the economy does not matter in this comparison. More specifically, if two allocations give exactly the same consumption streams to each member of cohorts of different generations, then they are equivalent by this optimality criterion, regardless of relative population sizes. This concept of optimality is also used by Nerlove, Razin and Sadka (1987). In section 8.2, I examine the Pareto optimality of competitive equilibrium.

8.1. Income tax transfer policy

Consider a policy which taxes each of the rich parents a lump sum amount $\tau > 0$ and distributes the revenues equally among the poor in such a way that the budget is balanced. We show in the following proposition that such a policy would imply a trade-off between intra-generational equity and inter-generational equity of income.

Proposition 8.1. A tax transfer from the rich to the poor at time t results in the following in period $t+1$:

(i) a net decrease in total output,
(ii) an increase in the wage gap of the two groups,
(iii) a decline in the capital stock, and
(iv) a net increase in the population, provided

$$(\alpha\sigma_2+\sigma_1)d_S/\alpha\sigma_2 d_U > L_t^U/L_t^S.$$

Proof. See appendix.

8.2. Child tax policy

Consider a policy which taxes parents for having unskilled children at the rate $\tau>0$ per child and invests the revenues prudently. The following proposition states the effects of such a policy.

Proposition 8.2.[6] *By taxing parents of any generation for having unskilled children and utilizing tax revenues prudently, one can increase total output, capital, and decrease total population in period $t+1$, without any decline in the tth period consumption of any parents, provided $\alpha<1-2\sigma_2/(\sigma_1+\sigma_2)$.*

Proof. See appendix.

Corollary 8.2.1. In a Cobb–Douglas economy, if voluntary transfer is small enough to satisfy $\alpha<1-2\sigma_2/(\sigma_1+\sigma_2)$, then the perfect foresight liquidity constrained equilibrium is not cohort-wise Pareto optimal, and the equilibrium follows a path of high population growth and low capital accumulation. In a similar way it could be shown that when capital markets are perfect the competitive equilibrium may not still be Pareto optimal.

Proof. Suppose that the economy is in steady state from time period t onwards. According to the above proposition, taxing parents for having unskilled children and subsidizing the skill and capital formation appropriately in period t would lead to a net increase in output and a decline in total population in the next period without a decline in anybody's consumption in period t. If we give now the skilled and unskilled labors their previous steady state wage rates, then the decentralized economy will continue to provide the

[6] This also indicates the possibility of Prisoners' dilemma in a decentralized economy.

same level of consumption to all members of generations $t+1$ onwards and yet in period $t+1$ there will be a net gain in output that could be distributed perhaps to the old people in period $t+1$. Hence, the perfect foresight equilibrium is not cohort-wise Pareto optimal. Q.E.D.

8.3. Social security tax transfer policies

While studies on social security programs in developed countries have attempted to measure its impact on private savings and labor supply, in developing countries such studies have investigated their effect on fertility rates only. The underlying hypothesis behind such effects on fertility has been that in the absence of well developed capital markets, parents depend on their children for old age support. However, a social security program in such less developed economies will clearly have synergistic effects on both fertility and savings rates of a household. Neither analytical nor empirical research has so far been directed to this important policy issue.

Consider a pay-as-you-go social security program of the following type. Each worker is taxed a fraction τ of his wage income in his working years. The cohort-wise actuarially fair benefits that are given to him in period $t+1$ when he retires are computed by the following formulae:

$$B^g_{t+1} = \tau W^g_{t+1} n^g_t \quad \text{for the group } g = \mathrm{S}, \mathrm{U}. \tag{8.1}$$

The parent's problems are as in (2.4) with $1-\alpha$ replaced by $(1-\alpha-\tau)$ in (2.4a) and the term B^g_{t+1} added to (2.4b). Note that the introduction of this program would not change the conditions for complete specialization for small $\tau > 0$.

Mimicking the computation of pre-tax equilibrium, we get the after-tax equilibrium quantities as follows: For $t \geqq 0$,

$$n^{\mathrm{U}}_t(\tau) = \frac{1-\alpha-\tau}{2d_{\mathrm{U}}} \cdot W^{\mathrm{U}}_t - \frac{B^{\mathrm{U}}_{t+1}}{2\alpha W^{\mathrm{U}}_{t+1}}.$$

Substituting the value of B^{U}_{t+1} from (8.1), we get

$$n^{\mathrm{U}}_t(\tau) = \left(1 + \frac{\tau}{2\alpha}\right)^{-1} \frac{1-\alpha-\tau}{2d_{\mathrm{U}}} \cdot W^{\mathrm{U}}_t. \tag{8.2}$$

Proceeding similarly for the skilled parents, we get

$$n^{\mathrm{S}}_t(\tau) = \left(1 + \frac{\tau\sigma_2}{2(\alpha\sigma_2 + \sigma_1)}\right)^{-1} \frac{\sigma_1(1-\alpha-\tau)}{(\alpha\sigma_2+\sigma_1)2d_{\mathrm{S}}} \cdot W^{\mathrm{S}}_t, \tag{8.3}$$

$$s_t(\tau)=\left(1+\frac{\tau\sigma_2}{2(\alpha\sigma_2+\sigma_1)}\right)^{-1}\frac{\sigma_1(1-\alpha-\tau)}{(\alpha\sigma_2+\sigma_1)2}\cdot W_t^{\mathrm{S}}. \tag{8.4}$$

Note that the after-tax capital–labor ratio, $k_{t+1}(\tau)$ in period $t+1$ is given by

$$k_{t+1}(\tau)=L_t^{\mathrm{S}}s_t(\tau)/[L_t^{\mathrm{S}}n_t^{\mathrm{S}}(\tau)+L_t^{\mathrm{U}}n_t^{\mathrm{U}}(\tau)]. \tag{8.5}$$

From (8.2)–(8.5) it could be easily shown that

$$\partial n_t^{\mathrm{S}}(\tau)/\partial\tau<0 \quad \partial n_t^{\mathrm{U}}(\tau)/\partial\tau<0, \quad \partial s_t(\tau)/\partial\tau<0, \quad \partial k_{t+1}(\tau)/\partial\tau<0.$$

Thus the following proposition has been proved.

Proposition 8.3. If a cohort-wise actuarially fair pay-as-you-go social security program is introduced in a Cobb–Douglas economy, then the family size of all parents will be smaller in that period, and overall capital–labor ratio will be higher in the next period.

9. Summary and conclusion

A structural explanation for the commonly observed quality–quantity trade-off and negative income–quantity relationships has been provided. It has been shown that in a Cobb–Douglas economy, such relationships hold only in the short-run, whereas in other economies, they may hold in the long-run also. A larger family size of the poor causes wider income differential for their children.

As a consequence of endogenous fertility, the equilibrium path is shown to converge to a steady state within a finite period. Therefore, a transitory policy will not have any long-run growth effects. The higher is the rate of voluntary transfers from children to parents, the lower will be the steady-state growth rate of population, capital stock and national income and also the lower will be the wage differential of the two groups in all periods.

General equilibrium effects of various income transfer policies are as follows: While a lump-sum tax transfer from the rich to the poor in any generation will cause a net increase in total population, with proportionately more unskilled labor, a decline in total output and a widening in the wage gap between the two groups, the taxation of parents for having unskilled children and the introduction of a pay-as-you-go social security program would reverse these effects. Moreover, the effect of income redistribution in any period is temporary.

If the two groups of households vary in their costs of investment in different assets, then there may not exist an equilibrium with perfect capital markets without government intervention. However, if capital markets are made perfect, the poor parents will reduce their family size, the rich parents will increase their family size and total investment in physical capital of both groups will be higher, and the wage differential of the two groups will be lower in all periods. However, the long-run growth effect will depend on the parameters of the economy. It has been also shown that the poor will gain and the rich will lose from perfection in the capital markets. Therefore the conflict of group interests may prevent the capital markets from being made perfect with private initiatives alone.

These results have important implications for population policies of developing countries. When the poor face higher costs of skill and capital formation than the rich, neither a policy of reducing income inequality by lump-sum tax transfers, nor a policy of subsidizing the cost of skill formation for the rich and poor uniformly will reduce population growth, let alone their adverse effects on capital and skill formations. An appropriate policy mix would be to build good quality schools, or to subsidize the education in rural areas, introduce a formal social security program, and provide high yielding risk-free investments and banking and insurance services to the poor.

Appendix

Proof of Proposition 3.2

Suppose $\eta_t^K = \eta_t^S = \eta_t^U = 0$, then by (2.6) we have

$$K_t^d = K_t^s, \qquad L_t^{Sd} = L_t^{Ss} \quad \text{and} \quad L_t^{Ud} = L_t^{Us}. \tag{A.1}$$

Note that

$$C_t^d = \sum_g (L_t^{gs} C_t^{g1} + L_{t-1}^{gs} C_t^{g2})$$

$$= \frac{1}{p_t} \cdot \left\{ \sum_g L_t^{gs} \left((1-\alpha) W_t^g - (\theta_K^g s_t^g + \theta_S^g n_t^{gS} + \theta_U^g n_t^{gU}) + \sum_g L_{t-1}^{gs} \right) q_t s_{t-1}^g \right.$$

$$\left. + a(W_t^S n_{t-1}^{gS} + W_t^U n_{t-1}^{gU}) + \gamma_t^g \pi_t \right\} \quad \text{from (2.4a) and (2.4b)}$$

$$=\frac{1}{p_t}\cdot\left(\sum_g W_t^g L_t^{gs}+q_t K_t^s+\pi_t-p_t\sum_g L_t^{gs}\{\theta_K^g s_t^g+\theta_S^g n_t^{gS}+\theta_U^g n_t^{gU}\}\right)$$

$$=C_t^s,\quad\text{for all } t\geqq 0 \text{ using (2.7) and (2.5).}\qquad\text{Q.E.D.}$$

Proof of Theorem 3.1

Assumption 3 implies that inequality in the budget constraints could be replaced by equality. By Assumptions 1 and 2, if an equilibrium exists, then for all $t\geqq 0$, $\rho_t=(p_t,q_t,W_t^S,W_t^U)>0$. For, suppose that any of them could be zero. Then while the supply of that commodity will be zero, its demand will be positive. Hence this could not be an equilibrium. By Proposition 3.2, the search for equilibrium prices, $\rho_t, t\geqq 0$ could be restricted to the simplex

$$A^M=\{(p,q,W^S,W^U)\geqq 0\,|\,p+q+W^S+W^U=1\}.$$

I now construct a sequence of equilibrium price vectors recursively. I will assume that agents within a group act identically. Thus one can treat each group of agents as a single agent. Define $p_0=1/(1+F_1+F_2+F_3)$, $q_0=p_0F_1$, $W_0^S=p_0F_2$, and $W_0^U=p_0F_3$, where, F_i's are the partial derivatives of F at the given initial stocks, K_0, L_0^S, L_0^U. Therefore, by construction, $\eta_0^K=\eta_0^S=\eta_0^U=0$. Hence by Proposition 3.2 $\eta_0^C=0$. Now suppose that equilibrium prices, ρ_t, and quantities K_t, L_t^S, and L_t^U are found for time periods $\leqq t$. We want to produce an equilibrium ρ_{t+1}. Let $\boldsymbol{a}_t^g$ be a solution of the problem of a representative parent of type g, $g=\mathrm{S,U}$, in time t. Define

$$A^g=\{a_t^g\,|\,\text{each element of } a_t^g\leqq M^g\},$$

where $M^g=\max\{W_t^g/p_t, M^*\}, g=\mathrm{S,U}$,

$$M^*=\max\{F(K_{t+1},L_{t+1}^S,L_{t+1}^U)\,|\,K_{t+1}+d_S L_{t+1}^S+d_U L_{t+1}^U$$
$$\leqq F(K_t,L_t^S,L_t^U)\}.$$

Let $A=A^U\times A^S\times A^M$.

Indeed in the above notations, a_t^U, $\boldsymbol{a}_t^S$, and $a_t^M(\equiv\rho_{t+1})$ are respectively the actions for the poor, rich, and the market; A^U, A^S, and A^M are their respective action spaces. Note that each action space is compact and convex.

For a given $\boldsymbol{a}$ in A, define the feasible sets of each agent by

$$\psi^U(a)=\{a_t^U\in A^U\,|\,a_t^U \text{ satisfies (2.4) for } g=\mathrm{U}\}\subset A^P,$$

$$\psi^{\mathrm{S}}(a)=\{a_t^{\mathrm{S}}\in A_R^{\mathrm{S}}\,|\,a_t^{\mathrm{S}} \text{ satisfies (2.3) for } g=\mathrm{S}\}\subset A^R,$$

$$\psi^{\mathrm{S}}(a)=\{a_t^{\mathrm{M}}\in A^{\mathrm{M}}\,|\,p_{t+1}=1/(1+F_1+F_2+F_3), q_{t+1}=p_{t+1}F_1,$$
$$W_{t+1}^{\mathrm{S}}=p_{t+1}F_2, W_{t+1}^{\mathrm{U}}=p_{t+1}F_3\}\subset A^{\mathrm{M}},$$

where F_i's are the partial derivatives of F at $K_{t+1}=\sum_g L_t^g S_t^g$,

$$L_{t+1}^{\mathrm{S}}=\sum L_t^g n_t^{g\mathrm{S}} \quad \text{and} \quad L_{t+1}^{\mathrm{P}}=\sum_g L_t^g n_t^{g\mathrm{U}}.$$

Lemma. ψ^{U}, ψ^{S}, *and* ψ^{M} *are continuous, convex-valued correspondences.*

Proof. It is easy to check that all the three correspondences are convex-valued. To prove that $\psi^{\mathrm{U}}(a)$ is continuous, I have to show that it is both upper hemicontinuous and lower hemicontinuous. First note that $\psi^{\mathrm{U}}(a)$ depends only on a^{M}. Let $\{a^{\mathrm{U}n}\}$ in A^{U}, and $\{a^{\mathrm{M}n}\}$ in A^{M} be two sequences such that for some $a^{\mathrm{U}0}$ in A^{U} and $a^{\mathrm{M}0}$ in A^{M}, $a^{\mathrm{U}n}\to a^{\mathrm{U}0}$, $a^{\mathrm{M}n}\to a^{\mathrm{M}0}$, and $a^{\mathrm{U}n}\in\psi^{\mathrm{U}}(a^{\mathrm{M}n})$, for all n. We want to show that $a^{\mathrm{U}0}\in\psi^{\mathrm{U}}(a^{\mathrm{M}0})$. Noting the fact that for any two real sequences, x_n, and y_n, $x_n\cdot y_n\to x\cdot y$ whenever $x_n\to x$ and $y_n\to y$, it is easy to note that $a^{\mathrm{U}0}$ does indeed satisfy the budget constraints in (2.4) when prices are $a_{\mathrm{M}0}$. Hence $a^{\mathrm{U}0}$ is in $\psi^{\mathrm{U}}(a^{\mathrm{M}0})$. Therefore, ψ^{U} is upper hemicontinuous.

Now let $\{a^{\mathrm{M}n}\}$ be a sequence in A^{M} such that $a^{\mathrm{M}n}\to a^{\mathrm{M}0}$ for some $a^{\mathrm{M}0}$ in A^{M} and let $a^{\mathrm{U}0}\in\psi^{\mathrm{U}}(b^{\mathrm{M}0})$. We want to show that there exists a sequence $\{a^{\mathrm{U}n}\}$ in A^{U} such that $a^{\mathrm{U}n}\to a^{\mathrm{U}0}$ and $a^{\mathrm{U}n}\in\psi^{\mathrm{U}}(a^{\mathrm{M}n})$. Define

$$a^{\mathrm{U}n}=(C_t^{\mathrm{U}10},(q_{t+1}^n s_t^{\mathrm{U}0}+\alpha W_{t+1}^{\mathrm{S}n}n_t^{\mathrm{USO}}+\alpha W_{t+1}^{\mathrm{U}n}n_t^{\mathrm{UUO}})/p_{t+1}^n, s_t^{\mathrm{U}0}, n_t^{\mathrm{USO}}, n_t^{\mathrm{UUO}}).$$

It is easy to check that indeed $a^{\mathrm{U}n}\in\psi^{\mathrm{U}}(a^{\mathrm{M}n})$ and $u^{\mathrm{U}n}\to a^{\mathrm{U}0}$. Hence, ψ^{U} is lower hemicontinuous.

The proof for ψ^{S} is similar.

To prove that $\psi^{\mathrm{M}}(a)$ is continuous, note that it depends only on a^{U} and a^{S}, and is the cross product of the following four continuous functions

$$\psi_1^{\mathrm{M}}=1/(1+F_1+F_2+F_3), \quad \text{and} \quad \psi_j^{\mathrm{M}}=F_{j-1}/(1+F_1+F_2+F_3),$$
$$j=2,3,4. \qquad \text{Q.E.D.}$$

Now define the optimal decision correspondences for each agent by

$$\mu^{\mathrm{U}}(a)=\{a_t^{\mathrm{U}} \text{ in } \psi^{\mathrm{U}}(a)\,|\,a_t^{\mathrm{U}} \text{ solves (2.4) for } g=\mathrm{U}\},$$

$$\mu^{\mathrm{S}}(a)=\{a_t^{\mathrm{S}} \text{ in } \psi^{\mathrm{S}}(a)\,|\,a_t^{\mathrm{S}} \text{ solves (2.4) with } g=\mathrm{S}\},$$

$$\mu^{\mathrm{M}}(a)=\psi^{\mathrm{M}}(a).$$

It is now evident that a fixed point of the correspondence μ from A to A defined by $\mu(a)=\mu^{\mathrm{U}}(a)\times\mu^{\mathrm{S}}(a)\times\mu^{\mathrm{M}}(a)$ is indeed an equilibrium. Since the utility functions are concave, μ is upper hemicontinuous by Debreu (1982, Theorem 3). Hence, by Kakutani's fixed point theorem there exists an equilibrium. Q.E.D.

Proof of Proposition 8.1

Suppose that each of the L_t^S rich parents are taxed by a lump-sum amount τ, and the tax receipts are distributed equally among the L_t^U poor parents. The after-tax equilibrium quantities are given by

$$n_t^U(\tau) = \beta^U(W_t^U + \tau r), \quad n_t^S(\tau) = \beta^S(W_t^S - \tau), \quad s_t(\tau) = \beta^K(W_t^S - \tau),$$

where $r = L_t^S/L_t^U$. Denote the after tax equilibrium output by $F(\tau)$. Then

$$F(\tau) = A(L_t^S s_t(\tau))^{\sigma_1}(L_t^S n_t^S(\tau))^{\sigma_2}(L_t^U n_t^U(\tau))^{\sigma_3}.$$

Note that

$$\begin{aligned}
\mathrm{d}\log F(\tau)/\mathrm{d}\tau|_{\tau=0} &= -(\sigma_1+\sigma_2)/W_t^S + \sigma_3 r/W_t^U \\
&= -(\sigma_1+\sigma_2)/W_t^S + \sigma_3 \cdot L_t^S \cdot W_t^S/(W_t^S \cdot L_t^U \cdot W_t^U) \\
&= -(\sigma_1+\sigma_2)/W_t^S + \sigma_3 \cdot \sigma_2/(W_t^S \cdot \sigma_3), \text{ as } L_t^S n_t^S = L_{t+1}^S \\
&= -\sigma_1/W_t^S < 0.
\end{aligned}$$

Hence (i) follows.

Let $h(\tau) = W_{t+1}^U(\tau)/W_{t+1}^S(\tau)$ be the after-tax wage ratio of the two groups in period $t+1$. Note that

$$h(\tau) = \frac{\sigma_3}{\sigma_2}\frac{L_t^S n_t^S(\tau)}{L_t^U n_t^U(\tau)}.$$

So,

$$\left.\frac{\mathrm{d}\log h(\tau)}{\mathrm{d}\tau}\right|_{\tau=0} = -\frac{1}{W_t^S} + \frac{r}{W_t^U} = \frac{1}{W_t^S}\left(\frac{L_t^S W_t^S}{L_t^U W_t^U} - 1\right) = \frac{1}{W_t^U}\left(\frac{\sigma_2}{\sigma_3} - 1\right).$$

Which is >0. Hence (ii) follows. To prove the rest note that

$$\left.\frac{\mathrm{d}n_t^U(\tau)}{\mathrm{d}\tau}\middle/\frac{\mathrm{d}n_t^S(\tau)}{\mathrm{d}\tau}\right|_{\tau=0} = -\frac{\sigma_2\alpha+\sigma_1}{\alpha\sigma_2}\cdot\frac{\mathrm{d_S}}{\mathrm{d_U}}\cdot r.$$

Whose absolute value is greater than one under the assumption of the proposition. It is trivial to show that $\mathrm{d}s_t(\tau)/\mathrm{d}\tau|_{\tau=0} < 0$. Hence (iii) and (iv) follow. Q.E.D.

Proof of Proposition 8.2

Suppose a tax rate of τ per unskilled child is imposed on generation t. Note that the after tax equilibrium quantity of unskilled children for each of the poor parents is given by

$$x_t^{\mathrm{U}}(\tau) = (1-\alpha)\, W_t^{\mathrm{U}}/(2d_{\mathrm{U}} + 2\tau).$$

So, the total tax revenue equals $\tau L_t^{\mathrm{U}} x_t^{\mathrm{U}}(\tau)$. To avoid complications in computation, I assume that this tax revenue is given equally to each of L_t^{S} parents in a lump-sum fashion. Then, the after-tax equilibrium quantity of skilled children and capital for each rich parent are given by

$$x_t^{\mathrm{U}}(\tau) = (W_t^{\mathrm{S}} + \tau n^{\mathrm{U}}(\tau) L_t^{\mathrm{U}}/L_t^{\mathrm{U}}) \quad \text{and} \quad s_t(\tau) = (W_t^{\mathrm{S}} + \tau n_t^{\mathrm{U}}(\tau) L_t^{\mathrm{U}}/L_t^{\mathrm{S}}).$$

Clearly this policy diminishes the first-period consumption of neither the poor nor the rich parents. Now I want to show that, in the after-tax new equilibrium in period $t+1$, the total output is larger and total population is smaller than the pre-tax equilibrium. To that end, note that the after-tax $(t+1)$th period equilibrium output is given by

$$F(\tau) = (L_t^{\mathrm{S}} s_t(\tau))^{\sigma_1} (L_t^{\mathrm{S}} n_t^{\mathrm{S}}(\tau))^{\sigma_2} (L_t^{\mathrm{U}} n_t^{\mathrm{U}}(\tau))^{\sigma_3}.$$

Therefore,

$$\begin{aligned} \mathrm{d}\log F(\tau)/\mathrm{d}\tau|_{\tau=0} &= (\sigma_1+\sigma_2)(1-\alpha)\, W_t^{\mathrm{U}} L_t^{\mathrm{U}}/(2d_{\mathrm{U}} W_t^{\mathrm{S}} \cdot L_t^{\mathrm{S}}) - \sigma_3/d_{\mathrm{U}} \\ &= (\sigma_1+\sigma_2)(1-\alpha)\sigma_3/(2d_{\mathrm{U}}\sigma_2) - \sigma_3/d_{\mathrm{U}} \\ &\gtreqless 0. \qquad \text{Q.E.D.} \end{aligned}$$

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